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Deformation of the cubic open string field theory

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ABSTRACT

We study a consistent deformation of the cubic open bosonic string theory in such a way that the non-planar world sheet diagrams of the perturbative string theory are mapped onto their equivalent planar diagrams of the light-cone string field theory with some length parameters fixed. An explicit evaluation of the cubic string vertex in the zero-slope limit yields the correct relationship between the string coupling constant and the Yang–Mills coupling constant. The deformed cubic open string field theory is shown to produce the non-Abelian Yang–Mills action in the zero-slope limit if it is defined on multiple D-branes. Applying the consistent deformation systematically to multi-string world sheet diagrams, we may be able to calculate scattering amplitudes with an arbitrary number of external open strings.

1. Introduction

If its perturbation theory is correctly defined, the covariant string field theory is expected to replace eventually the quantum field theory which has not been successful to describe quantum particles with spin two and higher spins. However, in practice, it is rather difficult to make use of the covariant cubic string field theory [1,2] to calculate the particle scattering amplitudes. The main reason is that the world sheet diagrams of cubic open string field theory are non-planar unlike those of the light-cone string field theory [3–9]. Witten [1] introduced an associative product between the open string field operators which represents the mid-point overlapping interaction. With the associative star product, the string field action takes the form of the Chern–Simons three-form which is invariant under the BRST gauge transformation. The cubic open string field theory has a merit of the BRST gauge invariance due to the associative algebra of the string field operators. But at the same time the mid-point overlapping interaction renders the world-sheet diagrams non-planar so that it becomes a difficult task to get the Fock space representations of the multi-string vertices.

The Fock space representation of the three-string vertex of the cubic open string field theory has been obtained by Gross and Jevicki in Refs. [10] and [11] by mapping the world-sheet diagram of six strings onto a circular disk and imposing an orbifold condition. The conformal mapping of the four-string world sheet to the upper half complex plane with branch cuts has been constructed by Giddings [12]. The Neumann functions of the three-string vertex have been calculated in Refs. [10,13,14] and the Neumann functions of the four-string vertex has been computed by Samuel in Ref. [15]. However, there seems to be no similarity between the conformal mappings for the three-string vertex and that of the four-string vertex. It seems also difficult to apply those constructions of the conformal mappings to more complex world sheet diagrams of multi-string vertices. Thus, it is desirable to develop a more systematic technique which could be applied to string scattering diagrams with an arbitrary number of external strings. In the present work, we propose a consistent deformation of the world sheet diagrams which transforms the non-planar diagrams of multi-string scattering into planar diagrams. Once having obtained the planar diagrams of the multi-string vertices, we can make use of the light-cone string field theory technique by mapping the world sheet diagrams onto the upper half complex plane. For the three-string vertex and the four-string vertex, it is enough to choose external string states such that physical string states are encoded only on the halves of the external strings. By an explicit calculation, we shall show that the deformed cubic string vertex yields the three-gauge field vertex with the correct Yang–Mills coupling constant in the zero-slope limit. The four-gauge field vertex of the Yang–Mills action shall be also evaluated by using the deformed world sheet diagram of the four-string vertex which is effectively generated by two cubic string vertices and an intermediate string propagator.

2. Deformation of the Witten's open string field theory diagrams

We shall begin with the Witten's cubic open string field theory action [1] on multi-D-branes which is given as

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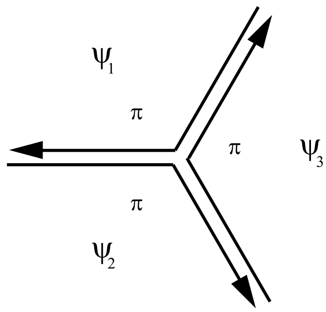

Fig. 1. The mid-point overlapping interaction of the cubic open string field theory.

$$\mathcal{S} = \int \mathrm{tr}\left(\Psi * Q\Psi + \frac{2g}{3}\Psi * \Psi * \Psi\right) \tag{1}$$

where Q is the BRST operator and the string field Ψ is $U(N)$ matrix valued

$$\Psi = \Psi^0 + \sum_a \Psi^a T^a, \quad a = 1, \cdots, N^2 - 1. \tag{2}$$

The star product between the string field operators is defined as follows

$$\begin{aligned}
(\Psi_1 * \Psi_2)[X^{(3)}(\sigma)] = \int & \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} DX^{(2)}(\sigma) \\
& \times \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \\
& \times \Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)], \\
X^{(3)}(\sigma) = & \begin{cases} X^{(1)}(\sigma) & \text{for } 0 \le \sigma \le \frac{\pi}{2}, \\ X^{(2)}(\sigma) & \text{for } \frac{\pi}{2} \le \sigma \le \pi. \end{cases}
\end{aligned} \tag{3}$$

In terms of the normal modes, the string coordinates $X^{(r)}(\sigma)$, $r = 1, 2, 3$ are expanded as

$$X^{(r)}(\sigma) = x^{(r)} + 2\sum_{n=1} \frac{1}{\sqrt{n}} x_n^{(r)} \cos(n\sigma). \tag{4}$$

It is the associativity of the star product algebra

$$(\Psi_1 * \Psi_2) * \Psi_3 = \Psi_1 * (\Psi_2 * \Psi_3) \tag{5}$$

that ensures invariance of the cubic string field theory action under the gauge transformation of the string field

$$\delta\Psi = Q * \epsilon + \Psi * \epsilon - \epsilon * \Psi. \tag{6}$$

In order to discuss the deformation of the cubic string field theory we extend the range of the world sheet coordinate σ as

$$0 \le \sigma \le \pi \implies 0 \le \sigma \le 2\pi. \tag{7}$$

The mid-point is now located at $\sigma = \pi$. Accordingly, the star product Eq. (3) and the normal mode expansion Eq. (4) should be appropriately redefined

$$X^{(r)}(\sigma) = x^{(r)} + 2\sum_{n=1} \frac{1}{\sqrt{n}} x_n^{(r)} \cos\left(\frac{n}{2}\sigma\right), \quad r = 1, 2, 3, \tag{8}$$

as shown in Fig. 1.

Fig. 2 depicts the world sheet diagram of three-string scattering. We observe that during the scattering process, physical information encoded on the left half of the first string and physical information encoded on the right half of the second string are not carried over to the third string. In view of scattering process roles

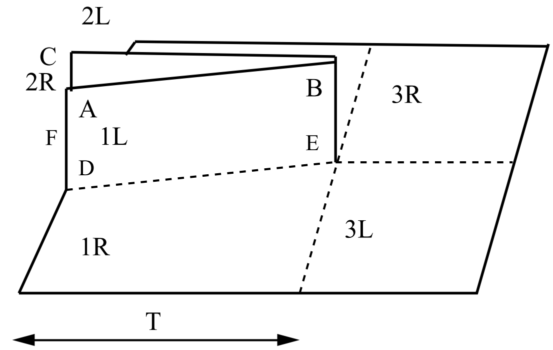

Fig. 2. The world sheet diagram of the three-string scattering.

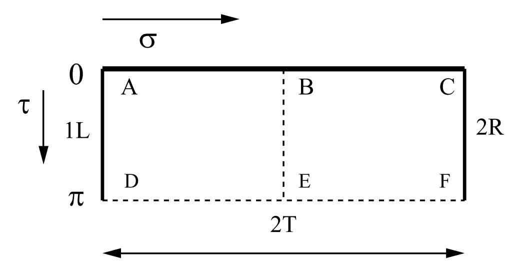

Fig. 3. Three-string vertex diagram of Witten's cubic open string field theory.

of the left half of the first string and the right half of the second string are auxiliary. Note that the strings satisfy the Neumann boundary condition on the boundary $\overline{ABC}$ in Fig. 1. We may separate the patch, corresponding to the world sheet trajectory of the left half of the first string and the right half of the second string from the rest part of the world sheet of three-string scattering. On the patch as we redefine the world sheet local coordinates by interchanging $\tau \leftrightarrow \sigma$, the boundary condition on $\overline{ABC}$ becomes

$$\partial_\sigma X^\mu = 0 \to \partial_\tau X^\mu = 0. \tag{9}$$

(See Fig. 3.) On the patch we also define new string coordinates

$$\begin{aligned}
X^\mu(\sigma) &= x^\mu + 2\sum_{n=1} \frac{1}{\sqrt{n}} x_n^\mu \cos\left(\frac{n\pi\sigma}{2T}\right) \\
&= x^\mu + \sum_{n=1} \frac{i}{\sqrt{n}}\left(a_n^\mu - a_n^{\mu\dagger}\right)\cos\left(\frac{n\pi\sigma}{2T}\right).
\end{aligned} \tag{10}$$

The Neumann condition on the boundary $\overline{ABC}$ may be written as

$$\partial_\tau X^\mu |N\rangle = 0, \tag{11}$$

$$|N\rangle = c_N \exp\left(-\frac{1}{2}\sum_{n=1} a_n^\dagger \cdot a_n^\dagger\right)|0\rangle, \tag{12}$$

where c_N is a normalization constant for the Neumann state. $|N\rangle$ is the open string analog of the Neumann boundary state of the closed string theory. The open string on the boundary $\overline{ABC}$ may propagate freely to the line $\overline{DEF}$ if the endpoints of the open string on the patch satisfy the Neumann condition

$$\partial_\sigma X^\mu(0) = 0 \text{ on } \overline{AD} \text{ and } \partial_\sigma X^\mu(\pi) = 0, \text{ on } \overline{CF}. \tag{13}$$

Then the open string state on $\overline{DEF}$ turns out to be the Neumann state again

$$\exp[-i\pi L_0]|N\rangle = -|N\rangle. \tag{14}$$