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Note on the butterfly effect in holographic superconductor models

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ABSTRACT

In this note we remark that the butterfly effect can be used to diagnose the phase transition of superconductivity in a holographic framework. Specifically, we compute the butterfly velocity in a charged black hole background as well as anisotropic backgrounds with Q-lattice structure. In both cases we find its derivative to the temperature is discontinuous at critical points. We also propose that the butterfly velocity can signalize the occurrence of thermal phase transition in general holographic models.

1. Introduction

Recently quantum butterfly effect has been becoming a hot spot of research which links the gauge/gravity duality to quantum many-body theory and quantum information theory [1–24]. Diagnosed by the out-of-time-order correlation (OTOC) functions, the butterfly effect describes the information scrambling over a quantum chaotic system. On gravity side, the butterfly effect is described by a shock wave geometry on the horizon that can be induced by an infalling particle which is exponentially accelerated. The butterfly effect ubiquitously exists in holographic theories due to its sole dependence on the near horizon data of the gravitational bulk theory. In particular, the Lyapunov exponent λ_L is always characterized by the Hawking temperature of the black hole as $\lambda_L = 2\pi k_B T$, while the butterfly velocity is completely determined by the horizon geometry [14,16,18]. Moreover, a bound on chaos is proposed as $\lambda_L \leqslant 2\pi k_B T$ and the saturation of this bound is viewed as the criterion for a quantum chaotic system to have a classical gravity dual description [9]. Stimulated by above investigation in holographic approach, many physicists in condensed matter as well as quantum information community have made great efforts in the measurement of the OTOC in laboratory [13,21–23]. The related progress is supposed to provide more practical tools to test the proposals in holographic theories, and in turn push forward the investigation on butterfly effects in quantum many-body systems.

In recent paper [18] we have investigated the butterfly effect in holographic models which exhibit metal-insulator transition (MIT) and found that the butterfly velocity v_B can diagnose quantum phase transitions (QPT). The key point on this is that the occurrence of QPT usually involves the RG flows from UV to different IR fixed points [25]. On the other hand, the butterfly velocity v_B depends on the IR geometry solely. Therefore, the change of IR fixed points may be reflected by the distinct behavior of v_B. In this note we intend to argue that the butterfly effect can exhibit attractive behavior during the course of thermal phase transition as well. This extension is natural, since based on Landau theory the occurrence of thermal phase transition is always accompanied by a symmetry breaking characterized by some order parameter. While in the context of holography, the spontaneous breaking of symmetry is usually a reflection of the instability of the background in bulk, signalized by the appearance of black hole hair which is supposed to deform the horizon, namely IR geometry strongly. Therefore, to provide evidence to support above argument we will investigate the temperature behavior of the butterfly velocity in holographic superconductor models. Specifically we will demonstrate that the derivative of v_B to the temperature is discontinuous at critical points of phase transition.

We organize this paper as follows. In next section we will first consider the butterfly effect in the simplest holographic model with superconductivity which is constructed over a charged black hole. Then we turn to study this effect over more complicated backgrounds with lattice structure in subsection 2.3. A brief discussion about possible extensions and experimental prospects will be presented in the end of this note.

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2. Butterfly effects and holographic superconductivity

In this section we will first introduce the butterfly velocity on anisotropic background. Using the anisotropic butterfly velocity results, we reveal that the butterfly velocity could diagnose superconductivity phase transitions.

2.1. Butterfly velocity on anisotropic background

Given a background with a black brane, we can compute the shockwave solution on the horizon generated by a particle freed at the asymptotic AdS region. The butterfly effect is represented by this sort of shockwave geometry, from which the Lyapunov exponent and butterfly velocity can be read off [14–16,18]. For a generic anisotropic black brane geometry,

$$ds^2 = \frac{1}{z^2}\Big[-(1-z)f(z)dt^2 + \frac{dz^2}{(1-z)f(z)} + V_x(z)dx^2 + V_y(z)dy^2\Big] \quad (1)$$

the butterfly velocity is also anisotropic, which can be written as [18]

$$\bar{v}_B(\theta) = v_B\sqrt{\frac{\sec^2(\theta)V_x(z)}{V_x(z)+\tan^2(\theta)V_y(z)}}\Bigg|_{z=1}, \quad (2)$$

where θ is the polar angle. $v_B = \bar{v}_B(0)$ is the butterfly velocity along the x-direction, given by

$$v_B = \sqrt{\frac{-2\pi\hat{T}V_y(z)}{V_y(z)[V_x'(z)-2V_x(z)]+V_x(z)[V_y'(z)-2V_y(z)]}}\Bigg|_{z=1}, \quad (3)$$

where the prime denotes the derivative to the radial coordinate z, and the $\hat{T}$ is the Hawking temperature of black brane (1). The metric ansatz (1), (2) and (3) are applicable for subsequent two holographic models in next two subsections.

Next, we investigate the butterfly effects in two holographic models involving phase transitions of superconductivity.

2.2. Butterfly effects in a simple holographic superconductor

The minimal ingredients to build a superconductor model in a holographic framework are provided by adding a charged complex scalar field into Einstein–Maxwell theory, in which the Lagrangian is [26,27]

$$\mathcal{L}_I = R + 6 - \frac{1}{4}F^2 - |D_\mu\Psi|^2 - M^2|\Psi|^2. \quad (4)$$

Notice that we have set the AdS radius $L=1$. $F=dA$ is the curvature of $U(1)$ gauge field A. Ψ is the charged complex scalar field with mass M and scaling dimension $\Delta_\Psi = 3/2 + (9/4+M^2)^{1/2}$, and the charge q. $D_\mu = \partial_\mu - iqA_\mu$ is the covariant derivative. We solve (4) by taking metric ansatz (1) and

$$A = \mu(1-z)a(z)dt, \qquad \Psi = \Psi(z), \quad (5)$$

where $f(z) \equiv (1+z+z^2-\mu^2z^3/4)S(z)$ and μ is the chemical potential in dual field theory. The Hawking temperature is then given by

$$\hat{T} = \frac{(12-\mu^2)S(1)}{16\pi}. \quad (6)$$

The corresponding dimensionless temperature is $T=\hat{T}/\mu$. Note that the system (4) is isotropic, the anisotropic metric (1) could be limited as isotropic case, *i.e.*, $V_x = V_y$. For simplicity, we set the mass and charge of the complex scalar field as $M^2=-2$ and $q=2$ such that its scaling dimension is $\Delta_\Psi = 2$ and its asymptotical behavior at UV is

$$\Psi = z\Psi_1 + z^2\Psi_2. \quad (7)$$

We shall treat Ψ_1 as the source and Ψ_2 as the expectation value of the dual operator. At the same time, we set $\Psi_1 = 0$ such that the condensation will take place spontaneously. At high temperature, the solution to equations of motion with ansatz (1) is simply the Reissner–Nordström AdS (RN-AdS) black brane solution with $\Psi(z)=0$ and $S(z)=a(z)=V_x(z)=V_y(z)=1$. However, below the critical temperature, imposing regular boundary conditions on the horizon and requiring the scalar field to decay at UV as in Eq. (7), one can numerically find new black brane solutions with scalar hair, which is dual to a superconducting phase in the boundary theory.

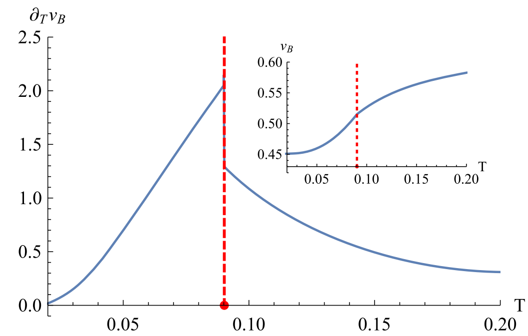

Fig. 1. Plot of $\partial_T v_B$ as a function of temperature T in the holographic superconductor model (4). The inset plot shows v_B as a function of T. The vertical dashed line denotes the superconducting phase transition temperature T_c, which is $T_c \simeq 0.0902$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

In this simple model with isotropy we have $V_x(z)=V_y(z)$ such that the formula is simplified as $v_B = \sqrt{\pi T\mu/[2V_x(1)-V_x'(1)]}$. Now we numerically compute v_B as the function of temperature T during the course of phase transition. Fig. 1 shows $\partial_T v_B$ as a function of temperature T and the inset plot shows v_B vs. T. In this figure it is evident that the derivative $\partial_T v_B$ is discontinuous at the critical temperature T_c (the red dashed line in vertical direction), which indicates that the butterfly velocity can be utilized as a new independent probe of the phase structure of the superconductor.

2.3. Butterfly effects in holographic Q-lattice superconductor

The second model we consider is the holographic superconductor on Q-lattices, which has been studied in [28]. Its Lagrangian reads as

$$\mathcal{L}_Q = R + 6 - \frac{1}{4}F^2 - |D_\mu\Psi|^2 - M^2|\Psi|^2 - |\nabla\Phi|^2 - m^2|\Phi|^2. \quad (8)$$

In comparison with the Lagrangian in (4), an additional neutral complex scalar field Φ with mass m is introduced to break the translational invariance [29]. We can solve this gravitational system by taking the metric (1) and

$$\Phi = e^{i\hat{k}x}z^{3-\Delta_\Phi}\phi(z), \qquad A = \mu(1-z)a(z)dt, \qquad \Psi = \Psi(z), \quad (9)$$

where $\Delta_\Phi = 3/2 + (9/4+m^2)^{1/2}$ is the scaling dimension of Φ. Note that since we only introduce the lattice in x-direction, our geometry is anisotropy. Now, each black brane solution is characterized by three scaling-invariant parameters, *i.e.*, the Hawking