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Heavy-quark spin symmetry partners of the $X(3872)$ revisited

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ABSTRACT

We revisit the consequences of the heavy-quark spin symmetry for the possible spin partners of the $X(3872)$. We confirm that, if the $X(3872)$ were a $D\bar{D}^*$ molecular state with the quantum numbers $J^{PC}=1^{++}$, then in the strict heavy-quark limit there should exist three more hadronic molecules degenerate with the $X(3872)$, with the quantum numbers 0^{++}, 1^{+-}, and 2^{++} in line with previous results reported in the literature. We demonstrate that this result is robust with respect to the inclusion of the one-pion exchange interaction between the D mesons. However, this is true only if all relevant partial waves as well as particle channels which are coupled via the pion-exchange potential are taken into account. Otherwise, the heavy-quark symmetry is destroyed even in the heavy-quark limit. Finally, we solve the coupled-channel problem in the 2^{++} channel with nonperturbative pions beyond the heavy-quark limit and, contrary to the findings of previous calculations with perturbative pions, find for the spin-2 partner of the $X(3872)$ a significant shift of the mass as well as a width of the order of 50 MeV.

1. Introduction

In the previous decade, lots of states were found experimentally in the heavy quarkonium mass range that did not at all fit into the scheme predicted by the until then very successful constituent quark model—for a review see, *e.g.*, Refs. [1,2]. Amongst those many states, the $X(3872)$ is special not only because it was the first such an extraordinary state discovered—it was first seen by the Belle Collaboration in 2003 [3]—but also because it resides extremely close to the $D^0\bar{D}^{*0}$ threshold. Indeed, with a mass $M_X = 3871.68 \pm 0.17$ MeV [4] its binding energy is as small as

$$E_X = m_0 + m_{*0} - M_X = 0.12 \pm 0.30\ \text{MeV}, \tag{1}$$

where m_0 (m_{*0}) denotes the mass of the D^0 (D^{*0}) meson [4]. Thus it has been regarded as one of the most promising candidates for a hadronic molecule, which may be either an S-wave bound state [5–10] or a virtual state in the $D\bar{D}^*$ system [11]; both possibilities are in line with its quantum numbers, which were determined by the LHCb Collaboration to be $J^{PC}=1^{++}$ [12].

Other models exist in addition to the hadronic molecule interpretation, which include $\chi_{c1}(2P)$ [13]—the first radial excitation of the P-wave charmonium $\chi_{c1}(1P)$,—a tetraquark [14], a mixture of an ordinary charmonium and a hadronic molecule [15,16], or a state generated in the coupled-channel dynamical scheme [17,18].

One of the celebrated theoretical tools used in studies of hadronic states with heavy quarks is the Heavy-Quark Spin Symmetry (HQSS). HQSS is based on the observation that for $\Lambda_{\rm QCD}/m_Q \to 0$, with m_Q denoting the quark mass, the strong interactions in the system are independent of the heavy quark spin. Then, although in case of the charm quark $\Lambda_{\rm QCD}/m_c \simeq 0.2$ is sizable and one expects non-negligible corrections to the strict symmetry limit, constraints from HQSS can still provide a valuable guidance also in the charm sector and in particular for the $X(3872)$ [19]. Meanwhile it was demonstrated in Ref. [20] that the consequences of HQSS are very different for the different scenarios for the X. It is therefore crucial to refine the quantitative predictions for the various scenarios. In this work we focus on a hypothesis that the $X(3872)$ is a molecular state and investigate the consequences that arise from HQSS as well as its leading violations. In particular, in Refs. [21–23] the spin partners of the isovector states $Z_b^+(10610)$ and $Z_b^+(10650)$ were investigated in

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the molecular picture and several degenerate states were predicted. Similarly, it was argued in Refs. [24,25] that one should expect a shallow bound state in the $D^*\bar{D}^*$ channel with the quantum numbers $J^{PC}=2^{++}$ — the molecular partner of the X(3872). In Ref. [26], based on an effective field theory with perturbative pions (X-EFT), the width of this state was estimated to be as small as a few MeV.

In all mentioned studies as well as in this work an assumption is made for the dominant molecule component of the wave functions of the states and observable implications of this assumption are investigated. In reality one can expect that there is an admixture of different components too. However, given the current quality of the data it appears unclear whether or not the effect of the subdominant components can be identified reliably within a given state. An exploratory study of the possible impact of genuine quarkonium states on the formation of the molecular spin multiplets is presented in Ref. [27]. In the future it would certainly be of interest to combine the insights presented in this paper with the ideas of Ref. [27].[1]

In this paper we refine further the implications of HQSS for the $X(3872)$ and its partners in the molecular picture and critically re-examine the findings of the above mentioned papers. In particular, we investigate in detail the implications of HQSS for the spin partners of the $X(3872)$ with and without one pion exchange (OPE). We adapt the methods of Ref. [23] to isoscalar states in the presence of OPE, with a special emphasis on the renormalisation to leading order in the heavy quark expansion. Furthermore, we go beyond the heavy-quark limit to demonstrate that the scales emerging in the coupled-channel approach due to the nonperturbative treatment of the pions generate a significant width of the 2^{++} spin partner of the $X(3872)$ as well as a sizeable shift of its mass.

2. Pionless theory—contact interactions only

2.1. Strict heavy-quark limit: spin partners of the $X(3872)$

Although pions play an important role in realistic calculations of the spin partners, as we shall demonstrate below, it is instructive to start from a simple analytically solvable model with the only S-wave contact interactions. The methods applied in this section to the isoscalar states in the charmonium sector are similar to those used in Ref. [23] for isovector states in the bottomonium sector. In this subsection we discuss the results at leading order (LO) in the heavy-quark expansion which we call the strict HQSS limit. In this case, the masses of the D and D^* are identical. The corrections due to the finite D^*-D mass splitting will be discussed in subsection 2.2.

The basis states introduced in Ref. [24] read

$$\begin{aligned}
0^{++}: &\quad \left\{D\bar{D}(^1S_0),\, D^*\bar{D}^*(^1S_0)\right\},\\
1^{+-}: &\quad \left\{D\bar{D}^*(^3S_1,-),\, D^*\bar{D}^*(^3S_1)\right\},\\
1^{++}: &\quad \left\{D\bar{D}^*(^3S_1,+)\right\},\\
2^{++}: &\quad \left\{D^*\bar{D}^*(^5S_2)\right\},
\end{aligned} \tag{2}$$

where the individual partial waves are labelled as $^{2S+1}L_J$ with S, L, and J denoting the total spin, the angular momentum, and the total momentum of the two-meson system, respectively. We define the C-parity eigenstates as

$$D\bar{D}^*(\pm) = \frac{1}{\sqrt{2}}\left(D\bar{D}^* \pm D^*\bar{D}\right), \tag{3}$$

which comply with the convention[2] for the C-parity transformation $\hat{C}\mathcal{M}=\bar{\mathcal{M}}$.

In this basis and for a given set of quantum numbers $\{JPC\}$, the leading-order EFT potentials $V_{\rm LO}^{(JPC)}$, which respect heavy-quark spin symmetry, read [24,30,31]

$$V_{\rm LO}^{(0++)} = \begin{pmatrix} C_{0a} & -\sqrt{3}C_{0b} \\ -\sqrt{3}C_{0b} & C_{0a}-2C_{0b}\end{pmatrix}, \tag{4}$$

$$V_{\rm LO}^{(1+-)} = \begin{pmatrix} C_{0a}-C_{0b} & 2C_{0b} \\ 2C_{0b} & C_{0a}-C_{0b}\end{pmatrix}, \tag{5}$$

$$V_{\rm LO}^{(1++)} = C_{0a}+C_{0b}, \tag{6}$$

$$V_{\rm LO}^{(2++)} = C_{0a}+C_{0b}, \tag{7}$$

where C_{0a} and C_{0b} are two independent low-energy constants.

The generic matrix integral equation for the scattering amplitude $a^{(JPC)}(p,p')$ reads

$$a^{(JPC)}(p,p') = V^{(JPC)}(p,p') - \int dk\, k^2\, V^{(JPC)}(p,k)G(k)a^{(JPC)}(k,p'), \tag{8}$$

and it simplifies considerably in the strict HQSS limit if only the leading-order contact interactions (4)–(7) are included. Here $G(k)$ denotes the matrix of the propagators of the heavy meson-antimeson pair in the intermediate state. In the single-channel case—see Eqs. (6) and (7)—it reads

$$G(k) = \frac{1}{k^2/\bar{m} - E - i0} \tag{9}$$

while for coupled channels—see Eqs. (4) and (5)—$G(k)$ is a 2×2 diagonal matrix with both nonzero elements given by Eq. (9). Here we used that in the strict HQSS limit the D^*- and D-meson masses m_* and m, respectively, coincide, $\bar{m}=m_*=m$. For the quantum numbers 1^{++} and 2^{++} Eq. (8) reduces to a single equation with the solution

$$a^{-1} = C_0^{-1} + \bar{m}\int dk \frac{k^2}{k^2-\bar{m}E - i0}, \tag{10}$$

where $C_0 = C_{0a}+C_{0b}$. The poles appear at the energies where the inverse amplitude, a^{-1}, vanishes. Accordingly, the value of the low-energy constant C_0 can be fixed from the binding energy of the $X(3872)$ (denoted below as E_X), used as input. Conversely, the binding energy in the 2^{++} channel, E_{X_2}, can be extracted from this equation, given that C_0 is known. Clearly, $E_{X_2}=E_X$ in the strict HQSS limit.

As shown in Refs. [19,21–23,32], in the heavy-quark limit, one can predict more states with the same binding energy. To this end, one can apply a unitary transformation [23], defined as

$$U = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \tag{11}$$

to the matrix bound-state Eq. (8) for the 0^{++} and 1^{+-} states. It is easy to see then that, taking $\phi=-\pi/6$ and $\phi=\pi/4$ for the 0^{++} and 1^{+-} potentials defined in Eqs. (4) and (5), respectively, one arrives for both quantum numbers at the diagonalised potential

[1] Note that according to Ref. [28] it might well be insufficient to include just a single quarkonium state in each channel.

[2] Notice that a different convention for the C-parity operator was used in Ref. [24]. As a consequence, the off-diagonal transitions of $V_{\rm LO}^{(0++)}$ in Ref. [24] have different sign as compared to Eq. (4), see also Sec. VI A in Ref. [29] for further details of our convention.