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Massive vector particles tunneling from black holes influenced by the generalized uncertainty principle

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ABSTRACT

This study considers the generalized uncertainty principle, which incorporates the central idea of large extra dimensions, to investigate the processes involved when massive spin-1 particles tunnel from Reissner–Nordstrom and Kerr black holes under the effects of quantum gravity. For the black hole, the quantum gravity correction decelerates the increase in temperature. Up to $\mathcal{O}(\frac{1}{M_f^2})$, the corrected temperatures are affected by the mass and angular momentum of the emitted vector bosons. In addition, the temperature of the Kerr black hole becomes uneven due to rotation. When the mass of the black hole approaches the order of the higher dimensional Planck mass M_f, it stops radiating and yields a black hole remnant.

1. Introduction

Hawking stated that black holes can release radiation thermodynamically due to quantum vacuum fluctuation effects near the event horizon [1]. Subsequently, Hawking radiation has attracted much attention from theoretical physicists and various methods have been proposed for deriving Hawking radiation. In particular, a semiclassical derivation was developed that models Hawking radiation as a tunneling process, which includes the null geodesic method and Hamilton–Jacobi method. The null geodesic method was first proposed by Kraus and Wilczek [2,3], and then developed further by Parikh and Wilczek [4–6]. The Hamilton–Jacobi method was proposed by Angheben *et al.* [7] as an extension of Padmanabhan's methods [8,9]. Both approaches to tunneling rely on the fact that the tunneling probability for the classically forbidden trajectory from inside to outside the horizon is given by $\Gamma = \exp(-2\mathrm{Im}I/\hbar)$, where I is the classical action of the trajectory. These two methods differ in how the imaginary part of the classical action is calculated. Many useful results have been obtained using the null geodesic and Hamilton–Jacobi methods [10–30].

A common feature of various quantum gravity theories, such as string theory, loop quantum gravity, and noncommutative geometry, is the existence of a minimum measurable length [31–34]. The generalized uncertainty principle (GUP) is a simple way of realizing this minimal length [35–37]. An effective model of the GUP in one-dimensional quantum mechanics, which incorporates the central idea of large extra dimensions, was given by [38]

$$L_f k(p) = \tanh\left(\frac{p}{M_f}\right), \tag{1}$$

$$L_f \omega(E) = \tanh\left(\frac{E}{M_f}\right), \tag{2}$$

where the generators of the translations in space and time are the wave vector k and the frequency ω, and L_f and M_f are the higher dimensional minimal length and Planck mass, respectively. L_f and M_f satisfy $L_f M_f = \hbar$. The quantization in position representation $\hat{x} = x$ leads to

$$k = -\mathrm{i}\partial_x, \quad \omega = +\mathrm{i}\partial_t. \tag{3}$$

Therefore, the low energy limit $p \ll M_f$ including the order of $(p/M_f)^3$ gives

$$p \approx -i\hbar\partial_x \left(1 - \beta\hbar^2\partial_x^2\right), \tag{4}$$

$$E \approx i\hbar\partial_t \left(1 - \beta\hbar^2\partial_t^2\right), \tag{5}$$

where $\beta = 1/(3M_f^2)$. Then, the modified commutation relation is given by

$$[x, p] = i\hbar\left(1 + \beta p^2\right), \tag{6}$$

and the generalized uncertainty relation (GUR) is

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$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\beta\langle p^2\rangle\right]. \tag{7}$$

From Eqs. (6) and (7), it can be concluded that the departure of GUP from the Heisenberg uncertainty principle increases with the momentum of the particle. We note that Eqs. (4)–(7) only apply to particles in the low energy limit $p \ll M_f$, which is the specific case considered in the present study. In the low energy regime, the parameter β should be constrained in experiments designed to test the uncertainty principle, such as those by [39,40]. Other generalized uncertainty relations can be found in previous studies. A widely discussed relation, $\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+l^2\frac{\Delta p^2}{\hbar^2}\right]$, was proposed based on some aspects of quantum gravity and string theory [35], where the cutoff l was selected as a string scale in the context of the perturbative string theory or Plank scale based on quantum gravity. Another interesting GUR was obtained by treating the mass source as a Gaussian wave function and the horizon as a horizon wave function [41], i.e., $\Delta r \simeq l_p\frac{m_p}{\Delta p}+\gamma l_p\frac{\Delta p}{m_p}$, where the first part represents the uncertainty of the radial size of the source and the second represents the horizon uncertainty, and γ is a parameter that represents the order of unity in the full quantum gravity regime, which becomes very small in the semiclassical regime.

Black holes are an important research area in the study of quantum gravity effects and many studies of black hole physics have incorporated the GUP. The thermodynamics of black holes have been investigated in the framework of GUP [42–48]. By combining the GUP with the tunneling method, Nozari and Mehdipour studied the modified tunneling rate of a Schwarzschild black hole [49]. The GUP-deformed Hamilton–Jacobi equation for fermions in curved spacetime was introduced and the corrected Hawking temperatures were derived for various types of spacetime in [50–58]. By studying the tunneling of fermions, it was found that the quantum gravity effects slowed down the increase in the Hawking temperatures, where this property naturally leads to a residual mass during black hole evaporation.

In this study, we investigate massive spin-1 particles ($W^{\pm}$, Z^0) tunneling across the horizons of black holes using the Hamilton–Jacobi method, which incorporates the minimal length effect via Eqs. (4) and (5). Our calculations show that the quantum gravity correction is related to the black hole's mass as well as to the mass and angular momentum of the emitted vector bosons. Furthermore, the quantum gravity correction explicitly retards the increase in temperature during the black hole evaporation process. As a result, the quantum correction will balance the traditional tendency for a temperature increase at some point during the evaporation, which leads to the existence of remnants.

The remainder of this paper is organized as follows. In Section 2, based on the GUP-corrected Lagrangian of the massive vector field, we derive the equation of motion for the vector bosons in curved spacetime. In Section 3, by incorporating GUP, we investigate the tunneling of charged massive bosons in a Reissner–Nordstrom black hole. The tunneling of massive bosons in a Kerr black hole is also studied and the remnants are derived in Section 4. Section 5 provides some discussion and the conclusions of this study. We use the spacelike metric signature convention $(-,+,+,+)$ in this study.

2. Generalized field equations for massive vector bosons

We start from the kinetic term of the uncharged vector boson field in flat spacetime within the framework of GUP, $\frac{1}{2}\widetilde{\mathfrak{B}}_{\mu\nu}\widetilde{\mathfrak{B}}^{\mu\nu}$, where the modified field strength tensor is given by

$$\widetilde{\mathfrak{B}}_{\mu\nu}=\left(1-\beta\hbar^2\partial_\mu^2\right)\partial_\mu\mathfrak{B}_\nu-\left(1-\beta\hbar^2\partial_\nu^2\right)\partial_\nu\mathfrak{B}_\mu. \tag{8}$$

It should be noted that additional derivative terms exist. Next, we generalize this to the case of a charged vector boson field ($W^{\pm}$) in charged black hole spacetime. Considering the gauge principle, the additional derivatives also act on the local unitary transformation operator $U(x)$, so they must also be replaced by covariant derivatives [59]:

$$\left(1-\beta\hbar^2\partial_0^2\right)\partial_0 \to \left(1+\beta\hbar^2 g^{00}D_0^{\pm 2}\right)D_0^{\pm}, \tag{9}$$

$$\left(1-\beta\hbar^2\partial_i^2\right)\partial_i \to \left(1-\beta\hbar^2 g^{ii}D_i^{\pm 2}\right)D_i^{\pm}, \tag{10}$$

where $D_\mu^{\pm}=\nabla_\mu \pm \frac{i}{\hbar}eA_\mu$ with ∇_μ is the geometrically covariant derivative, A_μ is the electromagnetic field of the black hole, and e denotes the charge of the W^+ boson. The difference in signs of the $\mathcal{O}(\beta)$ terms in Eqs. (9) and (10) is attributable to the fact that g^{00} always shares different signs with g^{ii}.

By defining

$$\mathcal{D}_0^{\pm}=\left(1+\beta\hbar^2 g^{00}D_0^{\pm 2}\right)D_0^{\pm} \text{ and } \mathcal{D}_i^{\pm}=\left(1-\beta\hbar^2 g^{ii}D_i^{\pm 2}\right)D_i^{\pm},$$

the GUP-corrected Lagrangian of W-boson field is given by

$$\begin{aligned}\mathcal{L}^{GUP}=&-\frac{1}{2}\left(\mathcal{D}_\mu^+W_\nu^+-\mathcal{D}_\nu^+W_\mu^+\right)\left(\mathcal{D}^{-\mu}W^{-\nu}-\mathcal{D}^{-\nu}W^{-\mu}\right)\\&-\frac{m_W^2}{\hbar^2}W_\mu^+W^{-\mu}-\frac{i}{\hbar}eF^{\mu\nu}W_\mu^+W_\nu^-,\end{aligned} \tag{11}$$

where $F_{\mu\nu}=\widehat{\nabla}_\mu A_\nu-\widehat{\nabla}_\nu A_\mu$, with $\widehat{\nabla}_0=\left(1+\beta\hbar^2 g^{00}{\nabla_0}^2\right)\nabla_0$ and $\widehat{\nabla}_i=\left(1-\beta\hbar^2 g^{ii}{\nabla_i}^2\right)\nabla_i$. Accordingly, the corresponding generalized action should be

$$\mathcal{S}^{GUP}=\int dx^4\sqrt{-g}\mathcal{L}^{GUP}\left(W_\mu^{\pm},\partial_\mu W_\nu^{\pm},\partial_\mu\partial_\rho W_\nu^{\pm},\partial_\mu\partial_\rho\partial_\lambda W_\nu^{\pm}\right). \tag{12}$$

This action is invariant under a local $U(1)$ gauge transformation, which does not refer to spacetime transformation.

By varying the action (12) with respect to the fields W^- and W^+, it follows immediately that

$$\begin{aligned}&\frac{\partial\mathcal{S}}{\partial W_\nu^-}-\partial_\mu\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu W_\nu^-\right)}+\partial_\mu\partial_\rho\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu\partial_\rho W_\nu^-\right)}\\&-\partial_\mu\partial_\rho\partial_\lambda\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu\partial_\rho\partial_\lambda W_\nu^-\right)}=0,\end{aligned} \tag{13}$$

$$\begin{aligned}&\frac{\partial\mathcal{S}}{\partial W_\nu^+}-\partial_\mu\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu W_\nu^+\right)}+\partial_\mu\partial_\rho\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu\partial_\rho W_\nu^+\right)}\\&-\partial_\mu\partial_\rho\partial_\lambda\frac{\partial\mathcal{S}}{\partial\left(\partial_\mu\partial_\rho\partial_\lambda W_\nu^+\right)}=0.\end{aligned} \tag{14}$$

Then, by substituting the GUP Lagrangian (11) in (13), we obtain

$$\begin{aligned}&\partial_\mu\left(\sqrt{-g}W^{+\mu\nu}\right)-3\beta\partial_0\left[\sqrt{-g}g^{00}\left(e^2{A_0}^2+i\hbar e\nabla_0A_0\right)W^{+0\nu}\right]\\&+3\beta\partial_i\left[\sqrt{-g}g^{ii}\left(e^2{A_i}^2+i\hbar e\nabla_iA_i\right)W^{+i\nu}\right]\\&+3\beta\partial_0\partial_0\left(\sqrt{-g}g^{00}i\hbar eA_0W^{+0\nu}\right)\\&-3\beta\partial_i\partial_i\left(\sqrt{-g}g^{ii}i\hbar eA_iW^{+i\nu}\right)+\beta\hbar^2\partial_0\partial_0\partial_0\left(\sqrt{-g}g^{00}W^{+0\nu}\right)\\&-\beta\hbar^2\partial_i\partial_i\partial_i\left(\sqrt{-g}g^{ii}W^{+i\nu}\right)+\sqrt{-g}\frac{i}{\hbar}eA_\mu W^{+\mu\nu}\\&-\sqrt{-g}\frac{m_W^2}{\hbar^2}W^{+\nu}-\sqrt{-g}\frac{i}{\hbar}eF^{\mu\nu}W_\mu^+\end{aligned}$$