Physics Letters B 763 (2016) 308–312

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Higher spin super-Cotton tensors and generalisations of the linear–chiral duality in three dimensions

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ARTICLE INFO

Article history:
Received 27 September 2016
Received in revised form 24 October 2016
Accepted 25 October 2016
Available online 2 November 2016
Editor: M. Cvetič

ABSTRACT

In three spacetime dimensions, (super)conformal geometry is controlled by the (super-)Cotton tensor. We present a new duality transformation for $\mathcal{N}$-extended supersymmetric theories formulated in terms of the linearised super-Cotton tensor or its higher spin extensions for the cases $\mathcal{N} = 2, 1, 0$. In the $\mathcal{N} = 2$ case, this transformation is a generalisation of the linear–chiral duality, which provides a dual description in terms of chiral superfields for general models of self-interacting $\mathcal{N} = 2$ vector multiplets in three dimensions and $\mathcal{N} = 1$ tensor multiplets in four dimensions. For superspin-1 (gravitino multiplet), superspin-3/2 (supergravity multiplet) and any higher superspin $s \geq 2$, the duality transformation relates a higher-derivative theory to one containing at most two derivatives at the component level. In the $\mathcal{N} = 1$ case, we introduce gauge prepotentials for higher spin superconformal gravity and construct the corresponding super-Cotton tensors, as well as the higher spin extensions of the linearised $\mathcal{N} = 1$ conformal supergravity action. Our $\mathcal{N} = 1$ duality transformation is a higher spin extension of the known superfield duality relating the massless $\mathcal{N} = 1$ vector and scalar multiplets. Our $\mathcal{N} = 0$ duality transformation is a higher spin extension of the vector-scalar duality.

1. Introduction

Recently there has been renewed interest in dualities in three-dimensional (3D) field theories [1–3]. In this paper we consider 3D duality transformations for theories involving higher spin analogues of the Cotton tensor or its supersymmetric generalisations – the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ super-Cotton tensors [4–6]. The specific feature of 3D conformal gravity is that its geometry can be formulated such that the Cotton tensor fully determines the algebra of covariant derivatives, see e.g. [6]. Similarly, in 3D $\mathcal{N}$-extended conformal supergravity formulated in conformal superspace [6], the corresponding superspace geometry is controlled by the super-Cotton tensor. In the $\mathcal{N} = 2$ case, our higher spin duality transformation may be thought of as a generalisation of the famous linear–chiral duality.

The linear–chiral duality [7,8] is of fundamental importance in supersymmetric field theory, supergravity and string theory, in particular in the context of supersymmetric nonlinear sigma models [8–10]. It provides a dual description in terms of chiral superfields for general models of self-interacting 3D $\mathcal{N} = 2$ vector multiplets or 4D $\mathcal{N} = 1$ tensor multiplets [7]. The only assumption for the duality to work is that the 3D $\mathcal{N} = 2$ vector multiplet or 4D $\mathcal{N} = 1$ tensor multiplet appears in the superfield Lagrangian, $L(W)$, only via its field strength W, which is a real linear superfield,

$$\bar{D}^2 W = 0 , \qquad W = \bar{W} \quad \Longrightarrow \quad D^2 W = 0 . \tag{1.1}$$

It is pertinent to recall the definition of the linear–chiral duality in the 3D $\mathcal{N} = 2$ case we are interested in. We start from a self-interacting vector multiplet model with action

$$S[W] = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, L(W) , \tag{1.2}$$

and associate with it the following first-order model

$$S[\mathcal{W}, \Psi, \bar{\Psi}] = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \left\{ L(\mathcal{W}) - (\Psi + \bar{\Psi})\mathcal{W} \right\} ,$$
$$\bar{D}_\alpha \Psi = 0 . \tag{1.3}$$

Here the dynamical variables are a real unconstrained superfield $\mathcal{W}$, a chiral scalar Ψ and its complex conjugate $\bar{\Psi}$. Varying $S[\mathcal{W}, \Psi, \bar{\Psi}]$ with respect to the Lagrange multiplier Ψ gives the equation of motion $\bar{D}^2\mathcal{W} = 0$, and hence $\mathcal{W} = W$. Then the second term on the right of $S[\mathcal{W}, \Psi, \bar{\Psi}]$ drops out, and we are back to the vector multiplet model (1.2). On the other hand, we can vary (1.3) with respect to $\mathcal{W}$ resulting in the equation of motion

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http://dx.doi.org/10.1016/j.physletb.2016.10.071

$$L'(\mathcal{W}) = \Psi + \bar{\Psi} \ . \qquad (1.4)$$

This equation allows us to express $\mathcal{W}$ as a function of Ψ and $\bar{\Psi}$, and then (1.3) turns into the dual action

$$S_{\rm D}[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta}\, L_{\rm D}(\Psi, \bar{\Psi}) \ . \qquad (1.5)$$

It remains to point out that the constraint (1.1) is solved in the 3D case by

$$W = \Delta H \ , \qquad \Delta = \frac{\rm i}{2} D^\alpha \bar{D}_\alpha \ , \qquad (1.6)$$

where the real prepotential H is defined modulo gauge transformations of the form

$$\delta H = \lambda + \bar{\lambda} \ , \qquad \bar{D}_\alpha \lambda = 0 \ . \qquad (1.7)$$

It is worth recalling one more type of duality that is naturally defined in the 3D $\mathcal{N}=1$ and 4D $\mathcal{N}=2$ cases, the so-called complex linear–chiral duality [11,12] (see also [13] for a review). It provides a dual description in terms of chiral superfields for general models of self-interacting complex linear superfields Γ and their conjugates $\bar{\Gamma}$. The complex linear–chiral duality plays a fundamental role in the context of off-shell supersymmetric sigma models with eight supercharges [10,14]. The complex linear superfield Γ is defined by the only constraint

$$\bar{D}^2 \Gamma = 0 \ . \qquad (1.8)$$

The complex linear–chiral duality works as follows. Consider a 3D $\mathcal{N}=2$ supersymmetric field with action

$$S[\Gamma, \bar{\Gamma}] = \int d^3x d^2\theta d^2\bar{\theta}\, L(\Gamma, \bar{\Gamma}) \ . \qquad (1.9)$$

We associate with it a first-oder action of the form

$$S[V, \bar{V}, \Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ L(V, \bar{V}) - \Psi V - \bar{\Psi} \bar{V} \right\} \ , \qquad \bar{D}_\alpha \Psi = 0 \ . \qquad (1.10)$$

Here the dynamical superfields comprise a complex unconstrained scalar V, a chiral scalar Ψ and their conjugates. Varying (1.10) with respect to the Lagrange multiplier Ψ gives $V = \Gamma$, and then the second term in (1.10) drops out as a consequence of the identities

$$\int d^3x d^2\theta d^2\bar{\theta}\, U = -\frac{1}{4} \int d^3x d^2\theta\, \bar{D}^2 U = -\frac{1}{4} \int d^3x d^2\bar{\theta}\, D^2 U \ , \qquad (1.11)$$

for any superfield U. As a result, the first-order action reduces to the original one, $S[\Gamma, \bar{\Gamma}]$. On the other hand, we can consider the equation of motion for V,

$$\frac{\partial}{\partial V} L(V, \bar{V}) = \Psi \ , \qquad (1.12)$$

and its conjugate. The latter equations allow us to express the auxiliary superfields V and $\bar{V}$ in terms of Ψ and $\bar{\Psi}$. Then (1.10) turns into the dual action

$$S_{\rm D}[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta}\, L_{\rm D}(\Psi, \bar{\Psi}) \ . \qquad (1.13)$$

It should be mentioned that the complex linear–chiral duality can also be introduced in the reverse order, by starting with a chiral model

$$S[\Psi, \bar{\Psi}] = \int d^3x d^2\theta d^2\bar{\theta}\, K(\Psi, \bar{\Psi}) \ , \qquad \bar{D}_\alpha \Psi = 0 \ , \qquad (1.14)$$

and then applying a Legendre transformation to $S[\Psi, \bar{\Psi}]$ in order to result in a model described by a complex linear superfield Γ and its conjugate $\bar{\Gamma}$. This makes use of the first-order action

$$S[U, \bar{U}, \Gamma, \bar{\Gamma}] = \int d^3x d^2\theta d^2\bar{\theta} \left\{ K(U, \bar{U}) - \Gamma U - \bar{\Gamma} \bar{U} \right\} \ , \qquad \bar{D}^2 \Gamma = 0 \ . \qquad (1.15)$$

In the 4D $\mathcal{N}=1$ case, the first-order action (1.15) with $K(U, \bar{U}) = U\bar{U}$ was considered for the first time by Zumino [20]. However, he did not realise the fact that this construction leads to a new off-shell description for the scalar multiplet, which was an observation made in [11,12].

The higher-spin generalisation of the complex linear–chiral duality has been given in [15–17,19]. For every integer $2s = 3, 4, \dots$, it relates the two off-shell formulations for the massless superspin-s multiplet in four dimensions constructed[1] in [15–17] and for the massive superspin-s multiplet in three dimensions presented in [19]. The goal of this paper is to give higher-spin generalisations of the 3D $\mathcal{N}=2$ linear–chiral duality in three dimensions and its $\mathcal{N}=1$ and $\mathcal{N}=0$ cousins.

This paper is organised as follows. The $\mathcal{N}=2$ duality transformation is presented in section 2. In section 3 we introduce gauge prepotentials for higher spin superconformal geometry, construct the corresponding super-Cotton tensors, and present the higher spin extension of the linearised $\mathcal{N}=1$ conformal supergravity action. Our $\mathcal{N}=1$ duality transformation is also described in this section. Finally, section 4 is devoted to the non-supersymmetric ($\mathcal{N}=0$) higher spin duality transformation.

2. $\mathcal{N}=2$ duality

Let n be a positive integer. We recall the higher-spin $\mathcal{N}=2$ superconformal field strength, $W_{\alpha(n)} = \bar{W}_{\alpha(n)}$, introduced in [19]

$$W_{\alpha_1 \dots \alpha_n}(H) := \frac{1}{2^{n-1}} \sum_{J=0}^{\lfloor n/2 \rfloor} \left\{ \binom{n}{2J} \Delta \Box^J \partial_{(\alpha_1}{}^{\beta_1} \dots \partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} H_{\alpha_{n-2J+1} \dots \alpha_n)\beta_1 \dots \beta_{n-2J}} + \binom{n}{2J+1} \Delta^2 \Box^J \partial_{(\alpha_1}{}^{\beta_1} \dots \partial_{\alpha_{n-2J-1}}{}^{\beta_{n-2J-1}} H_{\alpha_{n-2J} \dots \alpha_n)\beta_1 \dots \beta_{n-2J-1}} \right\} \ , \qquad (2.1)$$

where $\lfloor x \rfloor$ denotes the floor (or the integer part) of a number x. The field strength $W_{\alpha(n)}$ is a descendant of the real unconstrained prepotential $H_{\alpha(n)}$ defined modulo gauge transformations of the form

$$\delta H_{\alpha(n)} = g_{\alpha(n)} + \bar{g}_{\alpha(n)} \ , \qquad g_{\alpha_1 \dots \alpha_n} = \bar{D}_{(\alpha_1} L_{\alpha_2 \dots \alpha_n)} \ , \qquad (2.2a)$$

where the complex gauge parameter $g_{\alpha(n)}$ is an arbitrary longitudinal linear superfield,

$$\bar{D}_{(\alpha_1} g_{\alpha_2 \dots \alpha_{n+1})} = 0 \ . \qquad (2.2b)$$

The field strength is invariant under the gauge transformations (2.2),

$$\delta W_{\alpha(n)} = 0 \ , \qquad (2.3)$$

and obeys the Bianchi identity

$$\bar{D}^\beta W_{\beta\alpha_1 \dots \alpha_{n-1}} = 0 \quad \Longleftrightarrow \quad D^\beta W_{\beta\alpha_1 \dots \alpha_{n-1}} = 0 \ , \qquad (2.4)$$

which implies

[1] See [18] for a review of the models proposed in [15,16].