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Stability of the graviton Bose–Einstein condensate in the brane-world

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ABSTRACT

We consider a solution of the effective four-dimensional Einstein equations, obtained from the general relativistic Schwarzschild metric through the principle of Minimal Geometric Deformation (MGD). Since the brane tension can, in general, introduce new singularities on a relativistic Eötvös brane model in the MGD framework, we require the absence of observed singularities, in order to constrain the brane tension. We then study the corresponding Bose–Einstein condensate (BEC) gravitational system and determine the critical stability region of BEC MGD stellar configurations. Finally, the critical stellar densities are shown to be related with critical points of the information entropy.

1. Introduction

Several aspects of black hole physics have been recently studied, by considering black holes as Bose–Einstein condensates (BEC) of a large number N of weakly interacting, long-wavelength, gravitons close to a critical point [1–3]. This paradigm has the merit to directly interconnect black hole physics to the study of critical phenomena, where quantum effects are relevant at critical points, even for a macroscopic number N of particles [4]. Although black holes are non-perturbative gravitational objects, the effective quantum field theory of gravitons that describes them can still be weakly coupled, due to large collective effects [3,5,6]. Black hole features that cannot be recovered in a standard semiclassical approach of gravity may then be encoded by the quantum state of the critical BEC [7,8], with the semiclassical regime obtained as a particular limit for $N \to \infty$. Moreover, describing black holes by a condensate of long-wavelength gravitons generates a self-sustained system, whose size equals the standard Schwarzschild radius and the gravitons are maximally packed [1–3,7]. A quantum field-theoretical analysis also clarified the relation between the emerging geometry of spacetime and the quantum theory [9].

Brane-world models are effective five-dimensional (5D) phenomenological realisations of the Hořava–Witten domain wall solutions [10], when moduli effects, engendered from the remaining extra dimensions, may be disregarded [11,12]. The brane self-gravity is identified by the brane tension σ, and the effective four-dimensional (4D) geometry, due to a compact stellar distribution, can be achieved by a Minimal Geometric Deformation (MGD) of the standard Schwarzschild solution in General Relativity (GR) [13–17]. The MGD method ensures that this brane-world effective gravitational solution reduces to the standard Schwarzschild solution, in the limit of infinite brane tension $\sigma^{-1} \to 0$. Therefore the MGD is a framework that provides corrections to GR, controlled by a parameter ζ, that is a function of the stellar distribution effective radius and the brane tension.

Finally, we recall that a harmonic black hole model was recently introduced [18], which can be viewed as an explicit realisation of a BEC of gravitons, with a regular interior. The energy density in this model is obtained from a three-dimensional harmonic potential, "cut" around the horizon size in order to accommodate for the continuum spectrum of scattering modes, and the Hawking radiation. Afterwards, this model was ameliorated by instead considering the Pöschl–Teller potential [19], which naturally contains a continuum spectrum above the bound states, contrary to the harmonic oscillator.

We shall here employ this last model to study a MGD BEC black hole and analyse its critical stable density, from the point of view of the information entropy [20,21], and statistical mechanics [22]. The information entropy has been applied to a variety of settings, and the stability of self-gravitating compact objects was already reported in Refs. [20,23]. In particular, Newtonian polytropes, neutron stars, and boson stars were studied in Ref. [23]. The information entropy is well-known to measure the underlying

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shape complexity of spatially localised configurations [20,21]. The less information involved in the modes that comprise a physical system, the smaller entropic information is required to represent the same physical system. The energy density is the main ingredient to compute the information entropy. In this framework, the critical stable density of a BEC MGD black hole will be here studied, by relating the stellar distribution conditional entropy and its central critical density. In other words, the conditional entropy will be used to study the gravitational stability.

This work is organised as follows: we review the MGD procedure in Section 2, and the BEC description of a MGD black hole is employed to establish a bound for the brane tension of a Eötvös brane-world model in Section 3; Section 4 is devoted to establish the interplay between the critical point in the stellar stability and the critical point of the conditional entropy in a BEC MGD self-gravitating system scenario; finally, we comment on our findings in Section 5.

2. Minimal geometric deformation

The MGD approach is designed to produce brane-world corrections to standard GR solutions, hence it is a suitable method to obtain inhomogeneous, spherically symmetric, stellar distributions that are physically admissible in the brane-world [17,24]. For example, the bound $\sigma \gtrapprox 5\times 10^6$ MeV^4 for the brane tension was obtained from the MGD in Ref. [30]. The MGD was originally applied in order to deform the standard Schwarzschild solution [13,16,17] and describe the 4D geometry of a brane stellar distribution. Moreover, the MGD paved the way for interesting developments concerning 5D black string solutions of 5D Einstein equations [26] in Eötvös variable brane tension models [27,28].

The method relies on the effective Einstein equations on the brane [29],

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda\, g_{\mu\nu} - \tilde{T}_{\mu\nu} = 0\,, \tag{1}$$

where the effective energy–momentum tensor is given by

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} + \mathcal{E}_{\mu\nu} + \frac{1}{\sigma}\,S_{\mu\nu}\,, \tag{2}$$

which contains the usual stress tensor $T_{\mu\nu}$ of brane matter (with four-velocity u^μ), and the (non-local) Weyl and high energy Kaluza–Klein corrections $\mathcal{E}_{\mu\nu}$ and $S_{\mu\nu}$. The Weyl tensor can be further decomposed as

$$\mathcal{E}_{\mu\nu} = \frac{6}{\sigma}\left[\mathcal{U}\left(\frac{1}{3}h_{\mu\nu} + u_\mu\, u_\nu\right) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu}\, u_{\nu)}\right], \tag{3}$$

where $h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ denotes the induced spatial metric, $\mathcal{P}_{\mu\nu}$ is the anisotropic stress, $\mathcal{U}$ stands for the Weyl bulk scalar, and $\mathcal{Q}_\mu$ denotes the energy flux field.

One then considers the general spherically symmetric metric,

$$ds^2 = A(r)\,dt^2 - \frac{dr^2}{B(r)} - r^2\,d\Omega^2\,, \tag{4}$$

in the effective equations (1). Any deformation of this static metric, with respect to a GR solution, must be caused by 5D bulk effects, in a brane-world scenario. Particularly, the radial component outside a compact stellar distribution, of average radius $r = R$, turns out to be given by [16,17]

$$B_+(r) = 1 - \frac{2\,M}{r} + \zeta\, e^{-I}, \tag{5}$$

where

$$I(r) = \int_R^r \left(\frac{A A''}{A'^2} + \frac{A'^2}{A^2} - 1 + \frac{2A'}{rA} + \frac{1}{r^2}\right)\left(\frac{2}{r} + \frac{A'}{2A}\right)^{-1} d\bar{r}\,, \tag{6}$$

where primes denote derivatives with respect to r. The parameter ζ describes the deformation induced onto the vacuum by bulk effects, evaluated at the surface of the stellar distribution. Therefore, ζ contains all relevant information of a Weyl fluid on the brane [30]. The matching conditions with the inner star metric then determine the outer metric for $r > R$ [13,26]. In particular, if one considers the standard Schwarzschild metric, the deformed outer metric components read [16]

$$A_+(r) = 1 - \frac{2\,M}{r}\,, \tag{7a}$$

$$B_+(r) = \left(1 - \frac{2\,M}{r}\right)\left[1 + \zeta\,\frac{\ell}{r}\left(1 - \frac{3\,M}{2\,r}\right)^{-1}\right], \tag{7b}$$

where ℓ is a length given by[1]

$$\ell \equiv R\left(1 - \frac{2M}{R}\right)^{-1}\left(1 - \frac{3M}{2R}\right). \tag{8}$$

This metric has two event horizons where $B_+ = 0$: one is the usual Schwarzschild horizon, $r_s = 2\,M$, and the second horizon is at $r_2 = \frac{3M}{2} - \zeta\,\ell$. The expression of ζ was previously derived [13,16],

$$\zeta(\sigma, R) \approx -\frac{0.275}{R^2\,\sigma}\,, \tag{9}$$

and the GR limit $\zeta \sim \sigma^{-1} \to 0$ implies that $r_2 < r_s$. One can therefore conclude that the gravitational field around the compact star is weaker than in GR.

3. BEC and MGD: a brane tension bound

In order to study BEC black holes with the MGD methods, let us start from the Klein–Gordon equation for a scalar field Ψ [19]

$$\left\{\left[i\hbar\,\partial_t - V(\vec{x})\right]^2 + \hbar^2\,\nabla^2 - \left[\mu + S(\vec{x})\right]^2\right\}\Psi(t,\vec{x}) = 0\,, \tag{10}$$

where μ denotes the rest mass and one included the time-independent vector and scalar potentials $V(\vec{x})$ and $S(\vec{x})$. Writing $\Psi(t,\vec{x}) = e^{-i\,\varpi\, t/\hbar}\,\Psi(\vec{x})$ and assuming $S = V$ yield

$$\left[-\frac{\hbar^2}{2\,(\varpi + \mu)}\,\nabla^2 + V - \frac{1}{2}\,(\varpi - \mu)\right]\Psi(\vec{x}) = 0\,, \tag{11}$$

which is just a Schrödinger equation with $m = \varpi + \mu$, and $E = \frac{1}{2}(\varpi - \mu)$. It represents the relativistic dispersion relation $\varpi^2 = \hbar^2 k^2 + \mu^2$, and we shall in particular consider the spherically symmetric Pöschl–Teller potential [19]

$$V = -\frac{3\,\mu}{\cosh(\mu\, r/\hbar)}\,, \tag{12}$$

for which one can find explicit solutions for $\Psi = \Psi(r)$ and compute the corresponding energy density. In fact, this graviton BEC can be macroscopically modelled by an anisotropic fluid, with local energy–momentum tensor of the form

$$T^{\mu\nu} = \left(p_\parallel - p_\perp\right) v^\mu v^\nu + (\varepsilon + p_\perp)\, u^\mu u^\nu + p_\perp\, g^{\mu\nu}\,, \tag{13}$$

where $u^\mu\, u_\mu = -1 = -v^\mu\, v_\mu$, and $u^\mu\, v_\mu = 0$, ε is the energy density, $p_\perp$ and $p_\parallel$ are the pressures perpendicular and parallel to the

[1] The deformation around the star surface is negative, in order to prevent a negative pressure for a solid crust [24].