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Effect of scalar field mass on gravitating charged scalar solitons and black holes in a cavity

Supakchai Ponglertsakul, Elizabeth Winstanley *

Consortium for Fundamental Physics, School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom

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ABSTRACT

We study soliton and black hole solutions of Einstein charged scalar field theory in cavity. We examine the effect of introducing a scalar field mass on static, spherically symmetric solutions of the field equations. We focus particularly on the spaces of soliton and black hole solutions, as well as studying their stability under linear, spherically symmetric perturbations of the metric, electromagnetic field, and scalar field.

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1. Introduction

In the phenomenon of charge superradiance, a classical charged scalar field wave incident on a Reissner–Nordström black hole is scattered with a reflection coefficient of greater than unity if the frequency, ω, of the wave satisfies the inequality [1]

$$0 < \omega < q\Phi_h, \tag{1}$$

where q is the charge of the scalar field and Φ_h is the electrostatic potential at the event horizon of the black hole. By this process, the charged scalar field wave extracts some of the electrostatic energy of the black hole. If a charged scalar field wave satisfying (1) is trapped near the event horizon by a reflecting mirror of radius r_m, the wave can scatter repeatedly off the black hole, and is amplified each time it is reflected. This can lead to an instability (the "charged black hole bomb") where the amplitude of the wave grows exponentially with time [2–5], providing the scalar field charge q and mass μ satisfy the inequality [5]

$$\frac{q}{\mu} > \sqrt{\frac{\frac{r_m}{r_-} - 1}{\frac{r_m}{r_+} - 1}} > 1, \tag{2}$$

where r_+ and r_- are, respectively, the radius of the event horizon and inner horizon of the black hole. The inequality (2) ensures that the area of the event horizon increases as the scalar field evolves [2], and implies that for fixed q and μ, the mirror radius r_m must be sufficiently large for an instability to occur. Physically, the scalar field wave must extract more charge than mass from the black hole, so that the black hole evolves away from extremality.

What is the ultimate fate of this charged black hole bomb instability? To answer this question, it is necessary to go beyond the test-field limit and consider the back-reaction of the charged scalar field on the black hole geometry. Recently, we studied static, spherically symmetric, black hole [6] and soliton [7] solutions of Einstein charged scalar field theory in a cavity, in the case where the scalar field mass μ is set equal to zero. For both soliton and black hole solutions, the scalar field vanishes on the mirror. We examined the stability of these charged-scalar solitons and black holes by considering linear, spherically symmetric, perturbations of the metric, electromagnetic field, and massless charged scalar field. In the black hole case [6], we found that if the scalar field has no zeros between the event horizon and mirror, then the black holes appear to be stable. On the other hand, if the scalar field vanishes inside the mirror then the system is unstable. The situation for solitons is more complex [7]. Even if the scalar field has no zeros inside the mirror, there are some solitons which are unstable. The unstable solitons have small mirror radius and large values of the electrostatic potential at the origin.

* Corresponding author.
E-mail addresses: supakchai.p@gmail.com (S. Ponglertsakul), E.Winstanley@sheffield.ac.uk (E. Winstanley).

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In [6] we conjectured that the stable black holes with charged scalar field hair could be possible end-points of the charged black hole bomb instability. This conjecture has been tested recently [8,9] by evolving the fully coupled, time-dependent, spherically symmetric, Einstein–Maxwell–Klein–Gordon equations in a cavity. Starting from a Reissner–Nordström black hole in a cavity with a small charged scalar field perturbation, the system evolved to a hairy black hole in which some of the charge of the original black hole was transferred to the scalar field.

For a massless charged scalar field, the work of [9] confirms our conjecture in [6] – the ultimate fate of the charged black hole bomb is an equilibrium black hole with scalar field hair. However, in [8,9] a massive charged scalar field is also considered. In this paper we therefore study the effect of introducing a scalar field mass on the soliton and black hole solutions found in [6,7]. Our aim is to examine whether the end-points of the charged black hole bomb instability found in [8,9] correspond to stable equilibrium solutions of the Einstein–Maxwell–Klein–Gordon equations.

To this end, we begin in section 2 by introducing Einstein massive charged scalar field theory. We study numerical soliton and black hole solutions of the static, spherically symmetric field equations in section 3, paying particular attention to the effect of the scalar field mass on the phase space of solutions. The stability of the solutions is investigated in section 4, before our conclusions are presented in section 5.

2. Einstein massive charged scalar field theory

We consider a self-gravitating massive charged scalar field coupled to gravity and an electromagnetic field, and described by the action

$$S = \frac{1}{2}\int \sqrt{-g}\, d^4x \Bigg[R - \frac{1}{2} F_{ab}F^{ab} - g^{ab} D^*_{(a}\Phi^* D_{b)}\Phi - \mu^2 \Phi^*\Phi \Bigg] \tag{3}$$

where g is the metric determinant, R the Ricci scalar, $F_{ab} = \nabla_a A_b - \nabla_b A_a$ is the electromagnetic field (with electromagnetic potential A_a), Φ is the complex scalar field, Φ^* its complex conjugate and $D_a = \nabla_a - iqA_a$ with ∇_a the usual space-time covariant derivative. Round brackets in subscripts denote symmetrization of tensor indices. The scalar field charge is q and μ is the scalar field mass. We use units in which $8\pi G = 1 = c$ and metric signature $(-,+,+,+)$.

Varying the action (3) gives the Einstein–Maxwell–Klein–Gordon equations

$$G_{ab} = T^F_{ab} + T^\Phi_{ab}, \qquad \nabla_a F^{ab} = J^b, \qquad D_a D^a \Phi - \mu^2 \Phi = 0, \tag{4}$$

where the stress-energy tensor $T_{ab} = T^F_{ab} + T^\Phi_{ab}$ is given by

$$T^F_{ab} = F_{ac}F_b{}^c - \frac{1}{4} g_{ab} F_{cd}F^{cd},$$
$$T^\Phi_{ab} = D^*_{(a}\Phi^* D_{b)}\Phi - \frac{1}{2} g_{ab}\left[g^{cd} D^*_{(c}\Phi^* D_{d)}\Phi + \mu^2 \Phi^*\Phi \right], \tag{5}$$

and the current J^a is

$$J^a = \frac{iq}{2}\left[\Phi^* D^a \Phi - \Phi \left(D^a \Phi \right)^* \right]. \tag{6}$$

We consider static, spherically symmetric, solitons and black holes with metric ansatz

$$ds^2 = -f(r)h(r)dt^2 + f^{-1}(r)dr^2 + r^2\left[d\theta^2 + \sin^2\theta \, d\varphi^2 \right], \tag{7}$$

where the metric functions f and h depend only on the radial coordinate r. It is useful to define an additional metric function $m(r)$ by

$$f(r) = 1 - \frac{2m(r)}{r}. \tag{8}$$

By a suitable choice of gauge (see [6,7] for details), we can take the scalar field $\Phi = \phi(r)$ to be real and depend only on r. The electromagnetic gauge potential has a single non-zero component which depends only on r, namely $A_\mu = [A_0(r), 0, 0, 0]$. Defining a new quantity $E = A_0'$, the static field equations (4) generalize those in [6,7] to include a nonzero scalar field mass and take the form

$$h' = r\left(qA_0\phi f^{-1} \right)^2 + r h \phi'^2, \tag{9a}$$

$$E^2 + \mu^2 h \phi^2 = -\frac{2}{r}\left[f'h + \frac{1}{2} f h' + \frac{h}{r}(f-1) \right], \tag{9b}$$

$$0 = f A_0'' + \left(\frac{2f}{r} - \frac{fh'}{2h} \right) A_0' - q^2\phi^2 A_0, \tag{9c}$$

$$0 = f\phi'' + \left(\frac{2f}{r} + f' + \frac{fh'}{2h} \right)\phi' + \left(\frac{q^2 A_0^2}{fh} - \mu^2 \right)\phi. \tag{9d}$$

3. Soliton and black hole solutions

We now consider soliton and black hole solutions of the static field equations (9). In both cases we have a mirror at radius r_m, on which the scalar field must vanish, so that $\phi(r_m) = 0$. As in [7], here we consider only solutions where the scalar field has its first zero on the mirror, since it is shown in [6] that black hole solutions for which the scalar field has its second zero on the mirror are linearly unstable.

3.1. Solitons

In order for all physical quantities to be regular at the origin, the field variables have the following expansions for small r:

$$m = \left(\frac{\phi_0^2\left[a_0^2 q^2 + h_0\mu^2 \right]}{12 h_0} \right) r^3 + O(r^5),$$
$$h = h_0 + \left(\frac{q^2 a_0^2 \phi_0^2}{2} \right) r^2 + O(r^4),$$
$$A_0 = a_0 + \left(\frac{a_0 q^2 \phi_0^2}{6} \right) r^2 + O(r^4),$$
$$\phi = \phi_0 - \left(\frac{\phi_0\left[a_0^2 q^2 - h_0\mu^2 \right]}{6 h_0} \right) r^2 + O(r^4), \tag{10}$$

where ϕ_0, a_0 and h_0 are arbitrary constants. By rescaling the time coordinate (see [7] for details), we can set $h_0 = 1$ without loss of generality. A length rescaling [7] can then be used to fix the scalar field charge $q = 0.1$. For each value of the scalar field mass μ, soliton solutions are then parameterized by the two quantities a_0 and ϕ_0.

Scalar field profiles for some typical soliton solutions are shown in Fig. 1. From the expansions (10), it can be seen that if the scalar field mass vanishes, $\mu = 0$, and $\phi_0 > 0$ then close to the origin the scalar field is decreasing [7]. This is no longer necessarily the case when $\mu > 0$. For $\phi_0 > 0$ and $h_0 = 1$, if $|a_0| > \mu/q$ then the scalar field is decreasing close to the origin, and, for the numerical solutions investigated, it monotonically decreases to zero on the mirror. If $|a_0| < \mu/q$ then the scalar field is increasing close to the

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