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Lee–Wick black holes

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ABSTRACT

We derive and study an approximate static vacuum solution generated by a point-like source in a higher derivative gravitational theory with a pair of complex conjugate ghosts. The gravitational theory is local and characterized by a high derivative operator compatible with Lee–Wick unitarity. In particular, the tree-level two-point function only shows a pair of complex conjugate poles besides the massless spin two graviton. We show that singularity-free black holes exist when the mass of the source M exceeds a critical value $M_{\rm crit}$. For $M > M_{\rm crit}$ the spacetime structure is characterized by an outer event horizon and an inner Cauchy horizon, while for $M = M_{\rm crit}$ we have an extremal black hole with vanishing Hawking temperature. The evaporation process leads to a remnant that approaches the zero-temperature extremal black hole state in an infinite amount of time.

1. Introduction

The quantization problems of the Einstein–Hilbert action are well known. In the past 40 years, many authors have tried to quantize gravity by introducing modifications to Einstein's gravity. The first higher derivative theory of gravity dates back to quadratic gravity, which was proposed by Stelle in 1977 [1]. Stelle's theory is renormalizable and asymptotically free, but it is not unitary, having a massive ghost state in the spectrum.

A class of self-consistent quantum theories is represented by weakly non-local modifications of Einstein's gravity. These theories were first discussed by Krasnikov and Kuz'min [2,3], following previous work by Efimov [4]. A significant contribution to this line of research was later provided by Tomboulis, who proposed a whole class of weakly non-local super-renormalizable gauge and gravitational theories [5–7]. Recently, there have been a renewed interest in this class of gravity theories, which have been extensively studied to better understand their quantum properties [8,9]. In particular, a simple extension of [10] turns out to be completely finite at quantum level. Preliminary studies of black holes and gravitational collapse in these theories are reported in [11–18].

The weakly nonlocal theories are a quasi-polynomial extension of the higher derivative theories introduced and studied by Asorey, Lopez, Shapiro in [19]. Recently, Shapiro has pointed out that the quantum effective action of weakly nonlocal theories has likely an infinite number of complex conjugate poles [20]. Therefore, we decided to come back to the local higher derivatives theories (beyond Stelle's theory) with the special property of admitting the graviton field and only complex conjugate poles (no real poles) in the classical spectrum [20,21]. These theories are unitary in agreement with the Lee–Wick prescription [22–24].

In this paper, we focus on the minimal theory that fulfils the properties listed above. The action is [21,25–28]:

$$S = 2\kappa^{-2} \int d^4x \sqrt{|g|} \left[R + \alpha_g^2 \, G_{\mu\nu} \Box R^{\mu\nu} \right], \quad (1)$$

where $\kappa^2 = 16\pi G_{\rm N}$, $\alpha_g = 1/\Lambda^2$, and Λ is the UV scale of the theory. Λ is not necessarily equal to the Planck mass, but it may be expected of the same order as the Planck mass.

Looking at the exact equations of motion (EOM), we can immediately infer that all Ricci-flat spacetimes are exact solutions of the theory in vacuum [29]. However, when a point-like source is introduced on the right side of the EOM, the Schwarzschild, the Kerr, and other Ricci-flat spacetimes are no more exact solutions. Indeed, the Newtonian potential turns out to be regular and constant near $r = 0$ in any general higher derivative theory [28,30,31] (in nonlocal gravity we have a similar regular behaviour [32]). We thus expect a similar regular behaviour also for the exact black hole solutions, if any, when the EOM are solved in a non-empty spacetime. On the footprint of these results, in this paper we only

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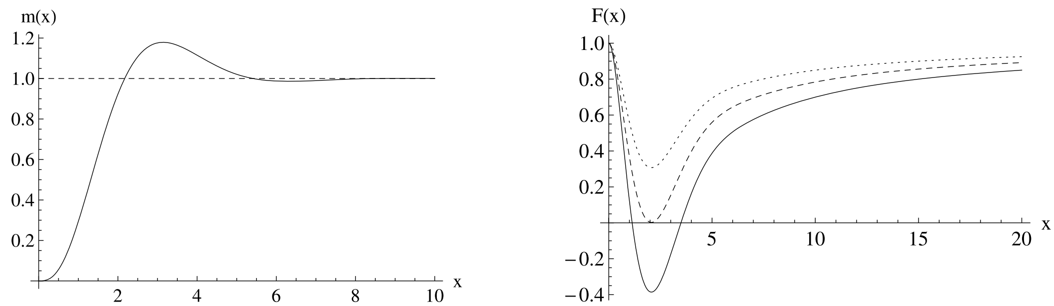

Fig. 1. Left panel: effective mass $m(x)$ for $M = 1$. Right panel: $F(x)$ for $M = 1.5$ (dotted line, no horizon), $M = 2.165$ (dashed line, one horizon), and $M = 3$ (solid line, two horizons). See the text for more details.

consider approximate EOM that we can somehow “solve exactly”, namely

$$\left(1+\frac{\Box^2}{\Lambda^4}\right) G_{\mu\nu} + O(R^2_{\mu\nu}) = 8\pi G_{\rm N} T_{\mu\nu}\,. \tag{2}$$

2. Black hole solutions

Let us consider a static point-like source of mass M. The only non-vanishing component of its energy-momentum tensor is:

$$T^0_0 = -M\delta(\vec{x})\,, \tag{3}$$

where $\delta(\vec{x})$ is the Dirac delta function and $\vec{x} = (x, y, z)$ are the Cartesian coordinates of the 3-space. Eq. (2) can be interpreted as the standard Einstein equations with an effective matter source on the left hand side. The effective energy-momentum tensor is

$$\tilde{T}_{\mu\nu} \approx \left(1+\frac{\Box^2}{\Lambda^4}\right)^{-1} T_{\mu\nu}\,. \tag{4}$$

The approximate EOM we are going to solve (leaving aside operators $O(R^2)$) read

$$G_{\mu\nu} = 8\pi G_{\rm N} \tilde{T}_{\mu\nu} = 8\pi G_{\rm N}\left(1+\frac{\Box^2}{\Lambda^4}\right)^{-1} T_{\mu\nu}. \tag{5}$$

We point out that this is a drastic approximation of the exact EOM coming from the theory (1), but the outcome will turn out to be consistent with the results obtained in the Newtonian approximation [28,30,31]. Moreover, as we have already pointed out, the Ricci-flat solutions are mathematically inconsistent in presence of a point-like source.

With the choice (3), the effective energy-momentum tensor can be written as

$$\tilde{T}^\mu_\nu = {\rm diag}\left(-\tilde{\rho}, \tilde{P}_r, \tilde{P}_\theta, \tilde{P}_\theta\right)\,, \tag{6}$$

where $\tilde{\rho}$ is the effective energy density, $\tilde{P}_r$ is the effective radial pressure, and $\tilde{P}_\theta$ is the effective tangential pressure. The effective energy density is

$$\begin{aligned}\tilde{\rho}(r) &= \left(1+\frac{\Box^2}{\Lambda^4}\right)^{-1} M\delta(\vec{x}) = M\int \frac{dk^3}{(2\pi)^3}\frac{e^{i\vec{k}\vec{x}}}{1+(k/\Lambda)^4} \\ &= \frac{M\Lambda^2}{4\pi r} e^{-\frac{r\Lambda}{\sqrt{2}}} \sin\frac{r\Lambda}{\sqrt{2}}\,.\end{aligned} \tag{7}$$

Let us assume that the static and spherically symmetric solution of Eq. (2) has the usual Schwarzschild-like form

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^2\,, \tag{8}$$

where

$$F(r) = 1 - \frac{2G_{\rm N} m(r)}{r}\,. \tag{9}$$

$m(r)$ is some effective mass and is a function of the radial coordinate r only because of the spherical symmetry.

With the energy-momentum tensor in Eq. (6) and the metric ansatz in (8), the Einstein equations (5) turn into

$$\frac{dm}{dr} = 4\pi r^2 \tilde{\rho}\,, \tag{10}$$

$$\frac{1}{F}\frac{dF}{dr} = \frac{2G_{\rm N}\left(m + 4\pi r^3 \tilde{P}_r\right)}{r\,(r-2G_{\rm N}m)}\,, \tag{11}$$

$$\frac{d\tilde{P}_r}{dr} = -\frac{1}{2F}\frac{dF}{dr}\left(\tilde{\rho}+\tilde{P}_r\right) + \frac{2}{r}\left(\tilde{P}_\theta - \tilde{P}_r\right)\,. \tag{12}$$

From Eq. (10), we find the function m

$$m(r) = 4\pi\int_0^r dx\, x^2 \tilde{\rho}(x)\,. \tag{13}$$

Eq. (11) is solved by $\tilde{P}_r = -\tilde{\rho}$, while From Eq. (12) we derive $\tilde{P}_\theta$.

If we plug the effective energy density (7) into Eq. (13) and we integrate over the radial coordinate r, we find

$$m(x) = Mf(x)\,, \tag{14}$$

where $x = \Lambda r/\sqrt{2}$ is a dimensionless coordinate and $f(x)$ is the dimensionless effective mass

$$f(x) = 1 - e^{-x}\left[(1+x)\cos x + x\sin x\right]\,. \tag{15}$$

The left panel in Fig. 1 shows the profile of $m(x)$ for $M = 1$. For $x \gg 1$, we recover the limit of general relativity with $m = M$ and the metric reduces to the Schwarzschild solution. At small radii, there are deviations from the classical picture. The characteristic length scale is $1/\Lambda$, which is the UV cut-off of the theory and is presumably extremely small, like the Planck length even if it is not necessarily the Planck length. $m(x)$ has a bump at $x \approx 3$ because the effective energy density is negative between $x \approx 3$ and $x \approx 5$. In other words, if we interpret this model as Einstein's gravity coupled to an effective energy-momentum tensor rather than as a non-local modification of Einstein's gravity, the effective energy-momentum tensor violates some energy conditions.

If we expand the effective mass around the centre $r = 0$, we have

$$m = \frac{\Lambda^3 r^3}{3\sqrt{2}} M + \ldots\,. \tag{16}$$

We thus find a de Sitter core