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Empirical moments of inertia of axially asymmetric nuclei

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ABSTRACT

Empirical moments of inertia, $\mathcal{J}_1$, $\mathcal{J}_2$, $\mathcal{J}_3$, of atomic nuclei with $E(4_1^+)/E(2_1^+) > 2.7$ are extracted from experimental $2^+_{g,\gamma}$ energies and electric quadrupole matrix elements, determined from multistep Coulomb excitation data, and the results are compared to expectations based on rigid and irrotational inertial flow. Only by having the signs of the $E2$ matrix elements, i.e., $\langle 2_g^+||\hat{M}(E2)||2_g^+\rangle$ and $\langle 0_g^+||\hat{M}(E2)||2_g^+\rangle\langle 2_g^+||\hat{M}(E2)||2_\gamma^+\rangle\langle 2_\gamma^+||\hat{M}(E2)||0_g^+\rangle$, can a unique solution to all three components of the inertia tensor of an asymmetric top be obtained. While the absolute moments of inertia fall between the rigid and irrotational values as expected, the relative moments of inertia appear to be qualitatively consistent with the $\beta^2\sin^2(\gamma)$ dependence of the Bohr Hamiltonian which originates from a SO(5) invariance. A better understanding of inertial flow is central to improving collective models, particularly hydrodynamic-based collective models. The results suggest that a better description of collective dynamics and inertial flow for atomic nuclei is needed. The inclusion of vorticity degrees of freedom may provide a path forward. This is the first report of empirical moments of inertia for all three axes and the results should challenge both collective and microscopic descriptions of inertial flow.

Atomic nuclei are finite many-body quantum systems composed of strongly interacting fermions that share remarkable similarities with other systems such as molecules, atomic clusters, and ultracold atomic gases. In particular, some of these quantum systems exhibit quenching of the moments of inertia from their rigid-body values at very low temperatures. For over half a century, superfluidity has been studied in both fermionic, e.g., atomic nuclei [1], and bosonic, e.g., liquid ^{4}He [2], systems. For fermionic systems, pairing is central to superfluidity. More recently, the nature of collective excitations and superfluidity of strongly interacting Fermi gases has been of active interest [3–9]; nearly perfect irrotational flow with a quadratic dependence on the deformation has been observed by Clancy et al. [6]. With these recent advances, the moments of inertia of atomic nuclei warrant an updated investigation.

The standard approach to evaluating the empirical moments of inertia of atomic nuclei has been to assume an axially symmetric rotor with rotational energies given by $E(I) = AI(I+1)$, where $A = \hbar^2/(2\mathcal{J})$ and $\mathcal{J}$ is the moment of inertia. For $I^\pi = 2^+$, the energy reduces to $E(2^+) = 6A$ and $\mathcal{J} = 3\hbar^2/E(2^+)$. A further assumption is that the first $I^\pi = 2^+$ state is unmixed with other states. This approach is sufficient to demonstrate that moments of inertia of atomic nuclei fall between the rigid-body and irrotational flow values, as shown by Bohr and Mottelson in 1955 [1]. However, this approach is limited in validating microscopic calculations of moments of inertia and in elucidating the existence of any underlying symmetries. A more thorough understanding of inertial flow requires knowledge of all three components of the inertia tensor; this requires input beyond the energy of the first excited 2^+ state.

The description of low-lying excited states of deformed even–even nuclei has been largely based on collective rotations and vibrations about the average β and γ quadrupole shape parameters (cf. Ref. [10] for a thorough overview). These nuclei possess rotational bands built on the 0^+ ground states and relatively low-lying excited 2^+ states, which could be the result of triaxial rotations or γ vibrations; distinguishing the two is notoriously difficult but the latter interpretation has been traditionally adopted. Fortunately, the Kumar–Cline sum rules [11] provide an experimental means for determining the average quadrupole deformation values and variances. These sum rules have demonstrated that the average γ deformations, $<\gamma>$, are non-zero; an axially symmetric nucleus would give zero. Unfortunately, the variances of the quadrupole deformations are not typically known; these are needed to differentiate between rigid and soft deformation. The few cases where

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the variances are known, e.g., the Os isotopes [12], lack precision but suggest that nuclei are neither rigid nor soft but somewhere in between.

We explore the implications of assuming β- and γ-rigid deformation (i.e., an axially asymmetric top) on the extracted moments of inertia. This is accomplished by using a recently formulated version of the triaxial rotor model with independent electric quadrupole and inertia tensors [13]; this is the simplest possible non-trivial view that allows a unique analytical solution to the three moments of inertia within the spin-2 subspace. While there have been investigations into the moments of inertia of axially asymmetric nuclei before, e.g., Refs. [14–18], empirical values for all three axes, to our knowledge, have never been reported.

In this Letter, empirical moments of inertia, $\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3$, of 12 atomic nuclei with $E(4_1^+)/E(2_1^+) > 2.7$ are extracted from experimental $2^+_{g,\gamma}$ energies and electric quadrupole matrix elements, and the results are compared to expectations based on rigid and irrotational inertial flow. The $E2$ matrix elements used in this study are from multiple-step Coulomb excitation data [12,19–26], most of which are from the past two decades. Only by having the signs of the $E2$ matrix elements, i.e., $\langle 2_g^+||\hat{M}(E2)||2_g^+\rangle$ and $\langle 0_g^+||\hat{M}(E2)||2_g^+\rangle\langle 2_g^+||\hat{M}(E2)||2_\gamma^+\rangle\langle 2_\gamma^+||\hat{M}(E2)||0_g^+\rangle$, can a unique solution to all three components of the inertia tensor be obtained.

The Hamiltonian for rotations about three axes (i.e., an asymmetric top) is

$$H = A_1\hat{I}_1^2 + A_2\hat{I}_2^2 + A_3\hat{I}_3^2, \tag{1}$$

where the parameters A_1, A_2, A_3 are related to the components of the inertia tensor by $A_1 = \hbar^2/(2\mathcal{J}_1)$, $A_2 = \hbar^2/(2\mathcal{J}_2)$, $A_3 = \hbar^2/(2\mathcal{J}_3)$ and $\hat{I}_1, \hat{I}_2, \hat{I}_3$ are the angular momentum operators in the body-fixed frame with a $|IK\rangle$ basis. The Hamiltonian can be rewritten as

$$H = A\hat{I}^2 + F\hat{I}_3^2 + G(\hat{I}_+^2 + \hat{I}_-^2), \tag{2}$$

where

$$A = \frac{1}{2}(A_1 + A_2),\ F = A_3 - A,\ G = \frac{1}{4}(A_1 - A_2), \tag{3}$$

and

$$\hat{I}_\pm = \hat{I}_1 \pm i\hat{I}_2. \tag{4}$$

When applied to doubly-even nuclei, there is an $I^\pi = 0^+$ ground state with $E(0^+) = 0$, no $I^\pi = 1^+$ state, and two mixed $I^\pi = 2^+$ states ($K^\pi = 0^+, 2^+$) with energies given by

$$H(2^+) = \begin{pmatrix} 6A & 4\sqrt{3}G \\ 4\sqrt{3}G & 6A + 4F \end{pmatrix}, \tag{5}$$

which yields

$$E(2^+) = 6A + 2F \pm 2\sqrt{F^2 + 12G^2}. \tag{6}$$

The mixing angle is related to G and F by

$$\tan 2\Gamma = 2\sqrt{3}\frac{G}{F} \tag{7}$$

(note, $\Gamma < 0$ because $G < 0$) and the resulting $E2$ matrix elements for the $I^\pi = 0^+, 2^+$ subspace are

$$\langle 0_g^+||\hat{M}(E2)||2_g^+\rangle = \sqrt{\frac{5}{16\pi}}Q_0\cos(\gamma + \Gamma), \tag{8}$$

$$\langle 0_g^+||\hat{M}(E2)||2_\gamma^+\rangle = \sqrt{\frac{5}{16\pi}}Q_0\sin(\gamma + \Gamma), \tag{9}$$

$$\langle 2_g^+||\hat{M}(E2)||2_\gamma^+\rangle = \sqrt{\frac{25}{56\pi}}Q_0\sin(\gamma - 2\Gamma), \tag{10}$$

and

$$\langle 2_g^+||\hat{M}(E2)||2_g^+\rangle = -\sqrt{\frac{25}{56\pi}}Q_0\cos(\gamma - 2\Gamma) = -\langle 2_\gamma^+||\hat{M}(E2)||2_\gamma^+\rangle. \tag{11}$$

The $E2$ matrix elements are described by three parameters, $Q_\circ$ (axial deformation), γ (axial asymmetry), and Γ (mixing angle). Further details can be found in Refs. [13,25,27–29]. While the 2^+ mixing angle, Γ, can be inferred from the excitation energies of higher spins, such an approach is not particularly sensitive and, more importantly, it does not lead to a unique empirical value.

Once the $Q_\circ$, γ, and Γ deformation and mixing parameters are determined from the experimental $E2$ matrix elements, the A, F, and G parameters of the Hamiltonian can be extracted exactly using the experimental 2^+ energies, viz.

$$F = \frac{E(2_\gamma^+) - E(2_g^+)}{4\sqrt{1 + \tan^2(2\Gamma)}}, \tag{12}$$

$$A = \frac{E(2_g^+) + E(2_\gamma^+) - 4F}{12}, \tag{13}$$

$$G = \frac{F}{2\sqrt{3}}\tan 2\Gamma, \tag{14}$$

where the empirical moments of inertia are

$$\mathcal{J}_1 = \frac{1}{2}\frac{\hbar^2}{A + 2G}, \tag{15}$$

$$\mathcal{J}_2 = \frac{1}{2}\frac{\hbar^2}{A - 2G}, \tag{16}$$

$$\mathcal{J}_3 = \frac{1}{2}\frac{\hbar^2}{A + F}. \tag{17}$$

It is important to stress that the signs of the $E2$ matrix elements are required to obtain a unique solution to all three components of the inertia tensor. In particular, $\langle 2_g^+||\hat{M}(E2)||2_g^+\rangle$ determines whether the electric quadrupole moment is prolate or oblate, and $\langle 0_g^+||\hat{M}(E2)||2_g^+\rangle\langle 2_g^+||\hat{M}(E2)||2_\gamma^+\rangle\langle 2_\gamma^+||\hat{M}(E2)||0_g^+\rangle$ determines whether $\gamma > |\Gamma|$ or $\gamma < |\Gamma|$.

The present results can be connected directly to results obtained using rigid and irrotational flow moments of inertia by

$$\mathcal{J}_{rigid,\,k} = B_{rigid}\left[1 - \sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - k\frac{2\pi}{3}\right)\right] \tag{18}$$

and

$$\mathcal{J}_{irrot.,\,k} = 4B_{irrot.}\beta^2\sin^2\left(\gamma - k\frac{2\pi}{3}\right), \tag{19}$$

where $k = 1, 2, 3$, $B_{rigid} = \frac{2}{5}MR^2 = 0.0138 \times A^{5/3}$ ($\hbar^2$/MeV), $B_{irrot.} = \frac{3}{8\pi}MR^2 = 0.00412 \times A^{5/3}$ ($\hbar^2$/MeV), $\beta = Q_\circ\sqrt{5\pi}/(3ZR^2)$, and $R = 1.2A^{1/3}$ (fm). It is important to highlight the fact that the irrotational-flow component of the moment of inertia in Eq. (19) resides in the mass parameter, $B_{irrot.}$. The $\beta^2\sin^2\left(\gamma - k\frac{2\pi}{3}\right)$ dependence is not explicitly limited to irrotational flow but results from the SO(5) invariance of the Bohr Hamiltonian (which happens to be fulfilled by irrotational flow), cf. page 121 of Ref. [10].

The mixing strength can be determined from the moments of inertia by

$$\Gamma = \frac{1}{2}\tan^{-1}\left(\sqrt{3}\frac{\mathcal{J}_2 - \mathcal{J}_1}{\frac{2\mathcal{J}_1\mathcal{J}_2}{\mathcal{J}_3} - \mathcal{J}_2 - \mathcal{J}_1}\right), \tag{20}$$