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Flavour breaking effects in the pseudoscalar meson decay constants

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ABSTRACT

The SU(3) flavour symmetry breaking expansion in up, down and strange quark masses is extended from hadron masses to meson decay constants. This allows a determination of the ratio of kaon to pion decay constants in QCD. Furthermore when using partially quenched valence quarks the expansion is such that SU(2) isospin breaking effects can also be determined. It is found that the lowest order SU(3) flavour symmetry breaking expansion (or Gell-Mann–Okubo expansion) works very well. Simulations are performed for $2+1$ flavours of clover fermions at four lattice spacings.

1. Introduction

One approach to determine the ratio $|V_{us}/V_{ud}|$ of Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, as suggested in [1], is by using the ratio of the experimentally determined pion and kaon leptonic decay rates

$$\frac{\Gamma(K^+\to\mu^+\nu_\mu)}{\Gamma(\pi^+\to\mu^+\nu_\mu)} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_{K^+}}{f_{\pi^+}}\right)^2 \frac{M_{K^+}}{M_{\pi^+}} \left(\frac{1-m_\mu^2/M_{K^+}^2}{1-m_\mu^2/M_{\pi^+}^2}\right)^2 (1+\delta_{\text{em}}) \qquad (1)$$

(where M_{K^+}, M_{π^+} and m_μ are the particle masses, and δ_{em} is an electromagnetic correction factor). This in turn requires the determination of the ratio of kaon to pion decays constants, f_{K^+}/f_{π^+}, a non-perturbative task, where the lattice approach to QCD may be of help. For some recent work see, for example, [2–10].

The QCD interaction is flavour-blind and so when neglecting electromagnetic and weak interactions, the only difference between the quark flavours comes from the mass matrix. In this article we want to examine how this constrains meson decay matrix elements once full $SU(3)$ flavour symmetry is broken, using the same methods as we used in [11,12] for hadron masses. In particular we shall consider pseudoscalar decay matrix elements and give an estimation for f_K/f_π and f_{K^+}/f_{π^+} (ignoring electromagnetic contributions).

2. Approach

In lattice simulations with three dynamical quarks there are many paths to approach the physical point where the quark masses take their physical values. The choice adopted here is to extrapolate from a point on the $SU(3)$ flavour symmetry line keeping the singlet quark mass $\overline{m}$ constant, as illustrated in the left panel of

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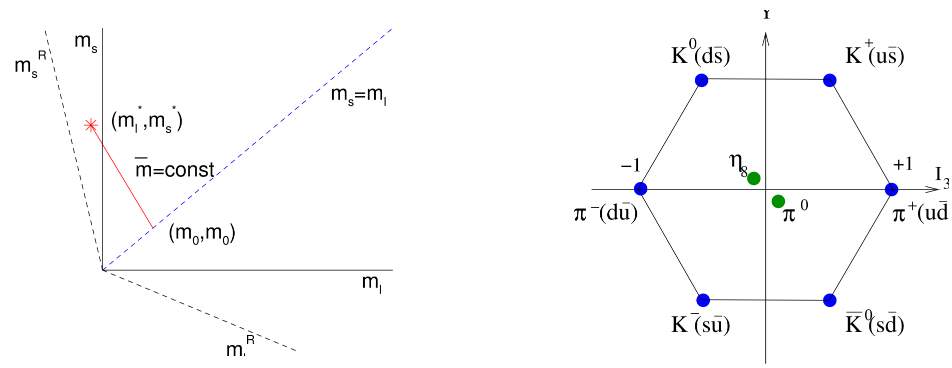

Fig. 1. LH panel: Sketch of the path for the case of two mass degenerate quarks, $m_u = m_d \equiv m_l$, from a point on the $SU(3)$ flavour symmetric line (m_0, m_0) to the physical point denoted with a $*$: (m_l^*, m_s^*). RH panel: The pseudoscalar octet meson.

Fig. 1, for the case of two mass degenerate quarks $m_u = m_d \equiv m_l$. This allows the development of an $SU(3)$ flavour symmetry breaking expansion for hadron masses and matrix elements, i.e. an expansion in

$$\delta m_q = m_q - \overline{m}\,, \quad \text{with} \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s) \tag{2}$$

(where numerically $\overline{m} = m_0$). From this definition we have the trivial constraint

$$\delta m_u + \delta m_d + \delta m_s = 0\,. \tag{3}$$

The path to the physical quark masses is called the 'unitary line' as we expand in the same masses for the sea and valence quarks. Note also that the expansion coefficients are functions of $\overline{m}$ only, which provided we keep $\overline{m} =$ const. reduces the number of allowed expansion coefficients considerably.

As an example of an $SU(3)$ flavour symmetry breaking expansion, [12], we consider the pseudoscalar masses, and find to next-to-leading-order, NLO, (i.e. $O((\delta m_q)^2)$)

$$\begin{aligned} M^2(a\bar{b}) = M_0^2 &+ \alpha(\delta m_a + \delta m_b) \\ &+ \beta_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &+ \beta_1(\delta m_a^2 + \delta m_b^2) + \beta_2(\delta m_a - \delta m_b)^2 \\ &+ \ldots\,, \end{aligned} \tag{4}$$

where m_a, m_b are quark masses with $a, b = u, d, s$. This describes the physical outer ring of the pseudoscalar meson octet (the right panel of Fig. 1). Numerically we can also in addition consider a fictitious particle, where $a = b = s$, which we call η_s. We have further extended the expansion to the next-to-next-to-leading or NNLO case, [13]. As the expressions start to become unwieldy, they have been relegated to Appendix A. (Octet baryons also have equivalent expansions, [13].)

The vacuum is a flavour singlet, so meson to vacuum matrix elements $\langle 0|\widehat{\mathcal{O}}|M\rangle$ are proportional to $1 \otimes 8 \otimes 8$ tensors, i.e. $8 \otimes 8$ matrices, where $\widehat{\mathcal{O}}$ is an octet operator. So the allowed mass dependence of the outer ring octet decay constants is similar to the allowed dependence of the octet masses. Thus we have

$$\begin{aligned} f(a\bar{b}) = F_0 &+ G(\delta m_a + \delta m_b) \\ &+ H_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + H_1(\delta m_a^2 + \delta m_b^2) \\ &+ H_2(\delta m_a - \delta m_b)^2 + \ldots\,. \end{aligned} \tag{5}$$

The $SU(3)$ flavour symmetric breaking expansion has the simple property that for any flavour singlet quantity, which we generically denote by $X_S \equiv X_S(m_u, m_d, m_s)$ then

$$X_S(\overline{m} + \delta m_u, \overline{m} + \delta m_d, \overline{m} + \delta m_s) = X_S(\overline{m}, \overline{m}, \overline{m}) + O((\delta m_q)^2)\,. \tag{6}$$

This is already encoded in the above pseudoscalar $SU(3)$ flavour symmetric breaking expansions, or more generally it can be shown, [11,12], that X_S has a stationary point about the $SU(3)$ flavour symmetric line.

Here we shall consider

$$\begin{aligned} X_\pi^2 &= \tfrac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\overline{K}^0}^2 + M_{K^-}^2)\,, \\ X_{f_\pi} &= \tfrac{1}{6}(f_{K^+} + f_{K^0} + f_{\pi^+} + f_{\pi^-} + f_{\overline{K}^0} + f_{K^-})\,. \end{aligned} \tag{7}$$

(The experimental value of X_π is ~ 410 MeV, which sets the unitary range.) There are, of course, many other possibilities such as $S = N$, Λ, Σ^*, Δ, ρ, r_0, t_0, w_0, [11,12,14].

As a further check, it can be shown that this property also holds using chiral perturbation theory. For example for mass degenerate u and d quark masses and assuming χPT is valid in the region of the $SU(3)$ flavour symmetric quark mass we find

$$X_{f_\pi} = f_0\left[1 + \frac{8}{f_0^2}(3L_4 + L_5)\overline{\chi} - 3L(\overline{\chi})\right] + O((\delta\chi_l)^2)\,, \tag{8}$$

where the expansion parameter is given by $\delta\chi_l = \overline{\chi} - \chi_l$ with $\overline{\chi} = \frac{1}{3}(2\chi_l + \chi_s)$, $\chi_l = B_0 m_l$, $\chi_s = B_0 m_s$, f_0 is the pion decay constant in the chiral limit, L_i are chiral constants and $L(\chi) = \chi/(4\pi f_0)^2 \times \ln(\chi/\Lambda_\chi^2)$ is the chiral logarithm. In eq. (8), as expected, there is an absence of a linear term $\propto \delta\chi_l$.

The unitary range is rather small so we introduce PQ or partially quenching (i.e. the valence quark masses can be different to the sea quark masses). This does not increase the number of expansion coefficients. Let us denote the valence quark masses by μ_q and the expansion parameter as $\delta\mu_q = \mu_q - \overline{m}$. Then we have

$$\begin{aligned} \tilde{M}^2(a\bar{b}) = 1 &+ \tilde{\alpha}(\delta\mu_a + \delta\mu_b) \\ &- (\tfrac{2}{3}\tilde{\beta}_1 + \tilde{\beta}_2)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &+ \tilde{\beta}_1(\delta\mu_a^2 + \delta\mu_b^2) + \tilde{\beta}_2(\delta\mu_a - \delta\mu_b)^2 \\ &+ \ldots\,, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \tilde{f}(a\bar{b}) = 1 &+ \tilde{G}(\delta\mu_a + \delta\mu_b) \\ &- (\tfrac{2}{3}\tilde{H}_1 + \tilde{H}_2)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &+ \tilde{H}_1(\delta\mu_a^2 + \delta\mu_b^2) + \tilde{H}_2(\delta\mu_a - \delta\mu_b)^2 \\ &+ \ldots\,, \end{aligned} \tag{10}$$