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Universality of DC electrical conductivity from holography

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ABSTRACT

We propose a universal formula of dc electrical conductivity in rotational- and translational-symmetries breaking systems via the holographic duality. This formula states that the ratio of the determinant of the dc electrical conductivities along any spatial directions to the black hole area density in zero-charge limit has a universal value. As explicit illustrations, we give several examples elucidating the validation of this formula: We construct an anisotropic black brane solution, which yields linear in temperature for the in-plane resistivity and insulating behavior for the out-of-plane resistivity; We also construct a spatially isotropic black brane solution that both the linear-T and quadratic-T contributions to the resistivity can be realized.

1. Introduction

The AdS/CFT correspondence provides a powerful tool to analyze strongly coupled systems, particularly for studying the transport properties of strongly coupled systems. One of the most famous results of the AdS/CFT applications, is the so-called Kovtun–Son–Starinets (KSS) bound $\eta/s \geq \hbar/(4\pi k_B)$, which states that for strongly coupled systems with a classical Einstein gravity dual description, the ratio of the shear viscosity η, to the entropy density s, obeys such a bound [1]. In most higher derivative gravity models, the bound is violated and there may still exist a lower bound [2,4,3,6,5,7], but even this is not clear [8].

Recently, the KSS conjecture was severely challenged by the anisotropic black brane systems, where the shear viscosity is a tensor and some components of the tensor can become considerably smaller, which parametrically violates the bound [9,10]. Considered a $d+1$-dimensional geometry with coordinates (t, x_i, z), and anisotropy only along the z-direction, the shear viscosity to entropy density ratio is related to the anisotropy as follows

$$\frac{\eta_{x_i z, x_i z}}{s} = \frac{\hbar}{4\pi k_B} \frac{g_{x_i x_i}}{g_{zz}}\bigg|_{r=r_{\rm H}}, \tag{1}$$

where $g_{x_i x_i}$ and g_{zz} are the line elements of the metric and $r_{\rm H}$ is the event horizon radius, respectively. For translational symmetry unbroken system, there is a universal relation between the graviton absorption cross-section and the black-brane horizon area in the large-incident-wavelength limit [11]: $\mathcal{A} = \Sigma(\omega = 0)$. The rise of the event horizon area-graviton cross-section equivalence is simply because the metric perturbation component $h^{x_j}_{x_i}$ satisfies the equation of motion of the minimally coupled massless scalar $\Box h^{x_j}_{x_i} = 0$. The spin-2 shear viscosity component is proportional to the graviton absorption cross-section via $\eta_{x_i x_j, x_i x_j} = \Sigma(\omega = 0)/2\kappa^2$. Therefore, the spin-2 shear viscosity component is linearly dependent on the event horizon area (i.e. the entropy density). However, for the spin-1 component $h^{x_i}_z$, the equation of motion is not identical to minimally coupled massless Klein-Gordon equation and thus the absorption cross-section of spin-1 vector field $h_{x_i z}$ in an anisotropic black-brane background is not equal to the black-brane horizon area. An arbitrary violation of the KSS bound would occur if $g_{x_i x_i}/g_{zz} \to 0$. In this anisotropic background, the rotational symmetry of the dual field theory is broken from $SO(d-1)$ to $SO(d-2)$. We thus have shear viscosities $\eta_{x_i z, x_i z}$, which are defined by the metric fluctuations $h_{x_i z}$. Such metric components carry spin 1 with respect to the $SO(d-2)$ symmetries [12]. Although the spin-2 components of the shear viscosity tensor in the $x_i - x_j$ plane satisfy the KSS bound, the shear force in the $x_i - z$ plane, which is related to the spin-1 metric components, can violates it. Furthermore, the diffusion bound $D \gtrsim C\hbar v_F^2/k_B T$ (C is a constant) will also break down [13–16]. The diffusivity bound was proposed to replace the Mott–Ioffe–Regel (MIR) bound in bad metals, and it is based on the KSS bound $\eta/s \geq C\hbar/k_B$ and the relation $\eta/s = DT/c^2$ for a vanishing chemical potential [14].

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We also notice that even in isotropic systems, the KSS conjecture can be violated due to momentum dissipation [17–21]. In translation invariance broken but isotropic systems, fluctuations of the metric components becomes massive and the corresponding shear viscosity does not yield a hydrodynamic description. In this case, the shear viscosity to entropy density ratio behaving as $\eta/s \sim T^{\nu}$ with ν a positive parameter, violating the KSS conjecture even in Einstein gravity. The shear viscosity now quantifies the rate of entropy production due to a strain.

One natural question is whether there is an alternative bound to be obeyed by the transport coefficients in such anisotropic systems. It is well-known that in condensed matter physics, it is notably universal that the materials are anisotropic with different properties in different directions. Remarkably, the transport in high-T_c cuprates is strongly two-dimensional in character and there is substantial anisotropy between the in- and out-of-plane (i.e. CuO_2 plane) resistivities. In contrast to the resistivity ρ_{ab} in the CuO_2 planes, where a generic behavior is observed to depend on the metallic temperature, the c-axis transport in high-temperature cuprates is very highly material-specific. Intriguingly, in most underdoped cuprates, $\rho_c(T)$ shows insulating behavior at all temperatures [22].

Therefore, the universal transporting properties of anisotropic systems deserve further studies. In this study, we will show that the ratio of the determinant of the electrical DC conductivities to the graviton absorption cross-section in anisotropic systems from holography in the zero-charges limit has a universal value

$$\frac{\prod_i \sigma_{ii}}{\mathcal{A}^{d-3}}\bigg|_{q_i=0} = \prod_i Z_i^{d-1}\bigg|_{r=r_{\rm H}}, \tag{2}$$

where $\mathcal{A}$ and Z_i are area density per unit volume of the black hole event horizon and gauge field couplings, respectively. In the minimal coupling case, $Z_i = 1$. Isotropic systems can be considered as special case of anisotropic systems.

The universal relation (2) is able to provide us some insights into the holographic realizations of the linear temperature resistivity:

1). For $Z(\phi) = 1$ and $d \geq 3$, isotropic black branes in the AdS space cannot be utilized to realize linear temperature resistivity in the zero-charges limit. Nevertheless, anisotropic black branes are good candidates in model-building of holographic strange metals.
2). For $d+1$-dimensional spatially isotropic Lifshitz black holes with $Z(\phi) = 1$ in the absence of hyperscaling violation, this relation indicates that $\sigma_{ii}|_{q_i=0} = [4\pi/(d+z-1)]^{d-3} T^{(d-3)/z}$, which is consistent with what obtained in Refs. [23,24] based on a universal scaling relation hypothesis: $\sigma(\omega = 0) = T^{(d-3)/z}\Theta(0)$, where z is a dynamical critical exponent and $\Theta(\omega)$ is a frequency dependent function.
3). This relation applies to shear viscosity-bound and electrical conductivity-bound violated systems, for example, systems considered in [20,25,26]. In [27], the authors conjectured that for the case $d = 3$, there exists a lower bound of dc electrical conductivity $\prod_i \sigma_{ii} > 1$. But it was soon found that this bound can be violated by a special coupling between the linear axion fields and the $U(1)$ gauge field [25,26].

The structure of this paper is organized as follows. In section 2, we present our main results by writing down the conductivity tensor in terms of horizon data for anisotropic systems. We then present three examples that reproduce particular features of strange metals in section 3. Discussions and conclusions are presented in section 4

2. Main results

Without loss of generality, we consider the Einstein–Maxwell-dilaton action with linear scalar fields

$$S = \int d^{d+1}x\sqrt{-g}\Bigg[\frac{1}{16\pi G}\Bigg(R - \frac{1}{2}\partial\phi^2 + V(\phi) - \frac{1}{2}\sum_{i=1}^{p-1} Y_i(\phi)\partial\psi_i^2\Bigg) - \frac{1}{4g_{d+1}^2}Z(\phi)F^2\Bigg]. \tag{3}$$

Hereafter, we select $16\pi G = g_{d+1}^2 = L = 1$, where L is the AdS radius, g_{d+1}^2 is the $d+1$-dimensional gauge coupling constant, and G is Newton's constant. Recently, this model has been widely studied in Refs. [28–35]. The solution to the above theory is assumed to be anisotropic

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + g_{xx}dx^j dx^j + g_{zz}dzdz, \tag{4}$$
$$\phi = \phi(r), \quad A = A_t(r)dt, \quad \psi_j = k_j x_j,\ j = 1\cdots d-2,$$
$$\psi_z = k_z z, \quad k_j \neq k_z.$$

The anisotropic direction is selected along the z-direction. We regard the $x_i - x_j$ plane as the "ab" plane and the z-direction as the "c"-axis in cuprates. The entropy density is given by $s = 4\pi(g_{xx}^{d-2}g_{zz})^{1/2}|_{r=r_{\rm H}}$. The electric charge density is given by $q \equiv -J^t = -\sqrt{-g}Z(\phi)\partial_r A_t$.

We impose a constant electric field in the x_i direction with magnitude E, which will generate electric currents only along the x_j direction. Let us consider a small perturbation in the black hole background

$$A_j = -Et + \delta a_{x_j}(r), \quad g_{tx_j} = \delta g_{tx_j}(r),$$
$$g_{rx_j} = g_{xx}\delta h_{rx_j}(r), \quad \psi = k_j x_j + \delta\chi_1. \tag{5}$$

From Maxwell equation $\partial_r(\sqrt{-g}Z(\phi)F^{rx_i}) = 0$, we can define a conserved current $J^{x_j} = -\sqrt{-g}g^{rr}g^{xx}Z(\phi)\partial_r a_{x_j} + \delta g_{tx_j}g^{xx}q$. In the absence of a charge density, we only have a contribution to the current from the gauge field $J^{x_j} \sim \partial_r a_{x_j}$. The conductivity can be determined based on the horizon regularity. In this case, we simply haves

$$\left(\sqrt{\frac{g_{tt}}{g_{rr}}}a'_{x_j}\right)' = 0. \tag{6}$$

Regularity at the horizon gives us

$$a_{x_j} = -\frac{E}{4\pi T}\ln(r - r_{\rm H}). \tag{7}$$

At finite charge density, we must know the behavior of δg_{tx_j} at the horizon. In the presence of momentum dissipation, δg_{tx_j} will take a finite value at the horizon

$$\delta g_{tx_j} = \frac{Eq}{k_j^2 Y_H g_{xx}^{\frac{d-3}{2}}}\Bigg|_{r=r_{\rm H}}, \tag{8}$$

where we use the notation $Y_H = Y(\phi_H)$ and $Z_H = Z(\phi_H)$. Therefore, the conserved current is obtained as

$$J^{x_j} = \left(g_{xx}^{\frac{d}{2}-2}g_{zz}^{\frac{1}{2}}Z_H E + \frac{Eq^2}{k_j^2 Y_H g_{xx}^{\frac{d-1}{2}}}\right)\Bigg|_{r=r_{\rm H}}. \tag{9}$$

Then, the DC conductivity is given by

$$\sigma_{jj} = \frac{J^{x_j}}{E} = \left(g_{xx}^{\frac{d}{2}-2}g_{zz}^{\frac{1}{2}}Z_H + \frac{q^2}{k_j^2 Y_H g_{xx}^{\frac{d-1}{2}}}\right)\Bigg|_{r=r_{\rm H}}. \tag{10}$$