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Clockwork inflation

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ABSTRACT

We investigate the recently proposed clockwork mechanism delivering light degrees of freedom with suppressed interactions and show, with various examples, that it can be efficiently implemented in inflationary scenarios to generate flat inflaton potentials and small density perturbations without fine-tunings. We also study the clockwork graviton in de Sitter and, interestingly, we find that the corresponding clockwork charge is site-dependent. As a consequence, the amount of tensor modes is generically suppressed with respect to the standard cases where the clockwork set-up is not adopted. This point can be made a virtue in resurrecting models of inflation which were supposed to be ruled out because of the excessive amount of tensor modes from inflation.

1. Introduction

The clockwork mechanism [1,2] allows to explain the presence of light degrees of freedom with highly suppressed interactions in theories where there are no small parameters to start with. A general theory of the clockwork mechanism valid for scalars, fermions, gauge bosons, and gravitons has been recently proposed in Ref. [3]. Let us briefly show how it operates for scalars and consider a theory endowed with a global $U(1)^{N+1}$ spontaneously broken at the scale f. The degrees of freedom at energies smaller than f are the $N+1$ Goldstone bosons π_i

$$U_i(x) = e^{i\pi_i(x)/f}, \quad i = 0, \cdots, N. \tag{1.1}$$

The π_i fields transform by a phase under the corresponding Abelian factor $U(1)_i$. Suppose now that the low-energy description of the theory is described by the Lagrangian

$$\begin{aligned}\mathcal{L} &= -\frac{f^2}{2}\sum_{i=0}^{N}\partial_\mu U_i^\dagger \partial^\mu U_i + \frac{m^2 f^2}{2}\sum_{i=0}^{N-1}\left(U_i^\dagger U_{i+1}^q + \text{h.c.}\right)\\ &= -\frac{1}{2}\sum_{i=0}^{N}(\partial\pi_i)^2 + \frac{m^2}{2}\sum_{i=0}^{N-1}(\pi_i - q\,\pi_{i+1})^2 + \mathcal{O}(\pi^4)\\ &= -\frac{1}{2}\sum_{i=0}^{N}(\partial\pi_i)^2 + \frac{1}{2}\sum_{i,j=0}^{N}\pi_i\, M^2_{\pi\;ij}\,\pi_j,\end{aligned} \tag{1.2}$$

where the presence of the explicit mass terms breaks softly the symmetry $U(1)^{N+1}$ down to a single U(1). The square mass matrix is given by

$$M_\pi^2 = m^2\begin{pmatrix} 1 & -q & 0 & \cdots & & 0\\ -q & 1+q^2 & -q & \cdots & & 0\\ 0 & -q & 1+q^2 & \cdots & & 0\\ \vdots & \vdots & \vdots & \ddots & & \vdots\\ & & & & 1+q^2 & -q\\ 0 & 0 & 0 & \cdots & -q & q^2\end{pmatrix}. \tag{1.3}$$

In the mass eigenstate basis ϕ_i $(i = 0, \ldots, N)$

$$\pi = O\,\phi, \qquad O^T M_\pi^2 O = \text{diag}\,(m^2_{\phi_0}, \ldots, m^2_{\phi_N}), \tag{1.4}$$

where O is a real orthogonal matrix, the eigenvalues are given by

$$\begin{aligned}&m^2_{\phi_0} = 0, \quad m^2_{\phi_k} = \lambda_k m^2,\\ &\lambda_k \equiv q^2 + 1 - 2q\cos\frac{k\pi}{N+1}, \qquad k = 1, \cdots, N.\end{aligned} \tag{1.5}$$

The elements of the rotation matrix O are given by

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$$O_{i0} = \frac{\mathcal{N}_0}{q^i}, \quad O_{ik} = \mathcal{N}_k \left[q \sin\frac{ik\pi}{N+1} - \sin\frac{(i+1)k\pi}{N+1} \right],$$
$$i = 0, .., N; \quad k = 1, \cdots, N, \tag{1.6}$$

and

$$\mathcal{N}_0 \equiv \sqrt{\frac{q^2-1}{q^2-q^{-2N}}}, \quad \mathcal{N}_k \equiv \sqrt{\frac{2}{(N+1)\lambda_k}}. \tag{1.7}$$

The key point of the clockwork mechanism is now that the massless eigenstate ϕ_0 is coupled to the rest of the fields in the theory with a coupling which is suppressed by $O_{i0} \sim q^{-i}$. In particular, if the rest of the degrees of freedom in the matter sector couples only to the N-th pion π_N, the state ϕ_0 couples to them with a suppressed coupling scaling like q^{-N}. If N is large and $q > 1$, then the coupling is efficiently suppressed.

In the case in which the number of copies is very large, it has been pointed out that there exists also a five-dimensional continuum limit of the clockwork mechanism [3]. It is achieved by introducing a dilaton field S in a five-dimensional braneworld with the fifth dimension compactified on $S_1/\mathbb{Z}_2$. The corresponding action reads

$$\mathcal{S} = \int d^4x dy \sqrt{-g} \left\{ \frac{M_5^3}{2} \left(R - \frac{1}{3}\partial_M S \partial^M S + 4k^2 e^{-\frac{2}{3}S} \right) \right.$$
$$\left. - \frac{e^{-\frac{1}{3}S}}{\sqrt{g_{55}}} [\delta(y)V_0 + \delta(y-\pi R)V_\pi] \right\}, \tag{1.8}$$

where k^2 characterizes the negative vacuum energy in the bulk, R is the radius of the fifth dimension, M_5 is the fundamental scale in the bulk, and V_0 and V_π are tensions on the brane satisfying the relation $V_0 = -V_\pi = -4kM_5^3$. The corresponding metric is found to be

$$ds^2 = e^{\frac{4}{3}k|y|} \left(dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right), \tag{1.9}$$

with $\eta_{\mu\nu}$ the flat Minkowski metric. In this picture hierarchies are produced on the $y = \pi R$ brane and the discrete suppression factor q^{-N} is replaced in the continuum with $e^{-k\pi R}$.

The goal of this note is to show that the clockwork mechanism can be adopted in inflationary theories to efficiently generate flat inflaton potentials sustaining a de Sitter phase as well as small masses and couplings to match the small amount of observed scalar perturbations. We will present in section 2 various examples of such ability from the four-dimensional discrete perspective. In section 3 we will study the phenomenon of inflation from the five-dimensional continuum perspective and show that the amount of clockworking producing small masses/couplings depends on the Hubble rate during inflation. Maybe more interestingly, in section 4 we show that, within the clockwork set-up, the clockwork charges of the gravitons are site-dependent and the amount of tensor modes generated during inflation is suppressed with respect to the standard scenario due to the fact that tensor modes are intrinsically bulk degrees of freedom. Section 5 contains our conclusions.

2. Clockwork inflation: the four-dimensional discrete perspective

In this section we show how to exploit the clockwork theory in inflation. The clockwork set-up is suitable to get either small masses (compared to the fundamental mass scale of the problem) or small couplings (compared to couplings of order unity). This is exactly what is needed during inflation in order to get the right amount of density fluctuations. The comoving curvature perturbation ζ in the flat gauge is [4]

$$\zeta = \left(\frac{H}{\dot\phi}\right)_* \delta\phi \sim \left(\frac{H}{M_{\rm pl}\sqrt{\epsilon}}\right)_*, \tag{2.1}$$

where the subscript $_*$ indicates that quantities should be computed at the epoch of Hubble radius exit for the comoving scale $k = aH$, ϕ is the inflaton field, H is the Hubble rate during inflation, $M_{\rm pl}$ is the reduced Planck mass, and one has to remember that observables scales in our current universe correspond to the last 60 e-folds or so before the end of inflation. Dots indicate differentiation with respect to time and

$$\epsilon = -\frac{\dot H}{H^2} \simeq \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \tag{2.2}$$

is one of the slow-roll parameters. The observed perturbations are matched if

$$\left(\frac{V^{3/2}}{M_{\rm pl}^3 V'}\right)_* \simeq 5.3 \cdot 10^{-4}. \tag{2.3}$$

2.1. Large field models of inflation

To illustrate the advantages of the clockwork set-up in producing flat potential during inflation, let us consider the class of large field models of inflation. The simplest model of inflation is given by a linear potential

$$V(\phi) = m^3\phi. \tag{2.4}$$

Of course, this is the potential during inflation and additional terms are supposed to be there in order to describe the dynamics after inflation, mainly reheating. For instance a simple modification of the potential above will be $V(\phi) = m^3(\phi^2 + am^2)^{1/2}$ where a is a constant and which gives the potential above for $\phi \gg m$, but has the right minimum at $\phi = 0$.

We know that the slow-roll conditions are attained when $\phi \gg M_{\rm pl}$ and that the density perturbations are given by [4]

$$\zeta \sim \left(\frac{m}{M_{\rm pl}}\right)^{3/2} \sim 10^{-5}, \tag{2.5}$$

for $m \sim 10^{15}$ GeV $\ll M_{\rm pl}$. In the clockwork scenario, we can assume that there are $N+1$ copies of the inflaton fields and the potential is

$$V(\pi_1, \cdots, \pi_N) = \frac{M_1^2}{2}\sum_{i=0}^{N-1}(\pi_i - q\,\pi_{i+1})^2 + M_2^3\pi_N, \tag{2.6}$$

where $M_2 \ll M_1$ (say smaller by a factor of 10), but both close to the fundamental scale. The first piece of the potential is invariant under a shift symmetry

$$\pi_i \to \pi_i + \frac{1}{q^i}, \tag{2.7}$$

which is broken by the last term in the potential. Upon diagonalization of the mass matrix (which is not altered by the presence of the linear term) and going to energy (to be identified with the Hubble rate H) much smaller than M_2, the lightest mass eigenstate state ϕ_0 will have a potential

$$V(\phi_0) = \frac{M_2^3}{q^N}\phi_0. \tag{2.8}$$

Taking for instance $M_2 \sim 10^{-1}M_{\rm pl}$, $q = 2$, we need $N \sim 20$ copies to match the observed level of perturbations. We should also note that possible one-loop contributions from the matter sector to the