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Quantum gravity and Standard-Model-like fermions

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ABSTRACT

We discover that chiral symmetry does not act as an infrared attractor of the renormalization group flow under the impact of quantum gravity fluctuations. Thus, observationally viable quantum gravity models must respect chiral symmetry. In our truncation, asymptotically safe gravity does, as a chiral fixed point exists. A second non-chiral fixed point with massive fermions provides a template for models with dark matter. This fixed point disappears for more than 10 fermions, suggesting that an asymptotically safe ultraviolet completion for the standard model plus gravity enforces chiral symmetry.

1. Introduction

An observationally viable model of quantum gravity must be compatible with the existence of matter and all its low-energy properties. This supplies observational tests for quantum gravity, as particular assumptions about quantum spacetime could be in conflict with low-energy observations. The specific fact that we focus on is the existence of light, chiral fermions. In the Standard Model, chiral symmetry forbids a microscopic fermion mass term $m_\psi \bar{\psi}\psi$. Fermion masses are generated from Yukawa interactions with the Higgs, and through chiral symmetry breaking in QCD for the quarks. Thus, fermion masses only emerge at scales far below the Planck scale. Here, we explore the interplay between quantum gravity and chiral symmetry, finding indications that chiral symmetry is a nontrivial observational constraint on models of quantum gravity: The fermion mass remains a Renormalization-Group (RG)-relevant coupling even under the impact of gravitational fluctuations. Thus, if chiral symmetry was broken above the Planck scale, it would not be restored automatically by the RG flow towards low energies, as it would for an irrelevant coupling. Accordingly, if chiral symmetry is broken in the ultraviolet (UV), the symmetry-violating effects are expected to generically grow towards low energies, typically leading to large fermion masses. As a specific illustration, we will focus on asymptotically safe quantum gravity [1], before analyzing models of quantum gravity from an effective-field-theory point of view.

2. Non-minimally coupled fermions in gravity

We analyze the RG scale dependence of the fermion mass under the impact of quantum-gravity fluctuations. We focus on its scaling dimension, which is RG relevant according to canonical counting in the free theory. If quantum fluctuations of spacetime cannot render it irrelevant, then it is expected to grow towards the infrared (IR). In this scenario, the appearance of chiral symmetry at low energies would either be impossible or require severe fine-tuning, unless the microscopic model of quantum gravity contained a mechanism to impose exact chiral symmetry. Crucially, the RG flow generates all terms that are compatible with the symmetries. Thus, once chiral symmetry is broken by a mass term, further non-chiral interactions are generated. Within the corresponding infinite-dimensional space of couplings, a sorting principle is provided by the canonical dimensionality of couplings. Perturbatively, only those couplings with vanishing or positive mass dimensionality can be relevant, i.e., can survive at low energies. Quantum-gravity effects could shift perturbatively slightly irrelevant couplings into relevance. For instance, in asymptotically safe gravity the relevant operators include several dimension-4-operators. Nonetheless, the departure from canonical scaling appears to be small and the canonical dimensionality remains a useful guiding principle, see, e.g., [2–6]. Thus, we analyze a truncated effective dynamics for N_f fermions containing all fermion bilinears with canonical dimensionality ≤ 5, i.e., including couplings with dimensionality ≥ -1

$$\Gamma_k = \Gamma_{\text{grav}} + iZ_\psi \int d^4x \sqrt{g}\, \bar{\psi}^i \nabla\!\!\!/ \psi^i + i\bar{m}_\psi \int d^4x \sqrt{g} \bar{\psi}^i \psi^i + i\bar{\xi} \int d^4x \sqrt{g} R \bar{\psi}^i \psi^i + i\bar{\zeta} \int d^4x \sqrt{g} \bar{\psi}^i \nabla^2 \psi^i. \quad (1)$$

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Table 1
We show the fixed-point values and critical exponents at the chiral non-Gaußian fixed point, as well as at the non-Gaußian fixed point which explicitly breaks chiral symmetry. For results in smaller truncations missing couplings and exponents are denoted by a "–".

Fixed point	Symmetry for fermions	g_*	λ_*	$m_{\psi *}$	ξ_*	ζ_*	η_ψ	θ_1	θ_2	θ_3	θ_4	θ_5
chiral non-Gaußian	chiral	2.52	−0.42	0	0	0	−0.17	3.54	1.34	0.84	−0.73	−1.27
chiral non-Gaußian	chiral	2.52	−0.42	0	–	–	−0.17	3.54	1.34	0.84	–	–
chiral non-Gaußian	chiral	2.52	−0.42	–	0	–	−0.17	3.54	1.34	–	−0.69	–
chiral non-Gaußian	chiral	2.52	−0.42	–	–	0	−0.17	3.54	1.34	–	–	−1.30
non-Gaußian	none	1.00	−0.27	1.01	1.10	−2.49	−0.56	3.65	1.66	0.59	−2.50 ± i 1.60	
non-Gaußian	none	2.52	−0.41	–	0.74	–	−0.15	3.54	1.37 ± i 0.04		–	–

Herein, Γ_k is the scale-dependent effective action that contains the effect of quantum fluctuations with momenta above k, only. In quantum gravity, the introduction of a scale relies on a dynamically generated background metric $\bar{g}_{\mu\nu}$. For the covariant derivative of the fermions we use the spinbase invariant formalism [7,8]. The kinetic term features a chiral $U(N_f)_L \times U(N_f)_R$ symmetry, under which left- and right-handed fermions transform separately. This symmetry is broken explicitly to a global $U(N_f)$ flavor symmetry by the mass term and the non-minimal interactions.

We employ the functional Renormalization Group [9] that is well-suited to trace fixed points away from the critical dimensionality of a given model and discover asymptotic safety, see, e.g., [10]. The Wetterich equation governs the momentum-scale dependence of the effective dynamics, encoded in Γ_k [9],

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr}\left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_t R_k \right], \tag{2}$$

with $\partial_t = k\partial_k$, see also [11,12]. In Eq. (2), $R_k(\Delta)$ is an IR regulator that provides a momentum-shell wise integration of the path integral: IR-modes for which the generalized "momentum" $\Delta < k^2$ are suppressed. The supertrace STr implements a summation/integration over the discrete/continuous eigenvalues of the field-dependent regularized propagator $(\Gamma_k^{(2)} + R_k)^{-1}$. The STr reduces to a sum over Lorentz and internal indices and a momentum integration for the case of a flat background, where $\Delta \to p^2$ (in the absence of gauge fields), with an additional negative sign for fermionic fields. For further details, see [20], for reviews [13–19] and specifically for gravity [21], following Reuter's seminal work [22].

To set up the RG flow for gravity, we use the background field method [23] and split the metric into background and fluctuation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{16\pi G}\, h_{\mu\nu}. \tag{3}$$

We work with the Einstein–Hilbert action

$$\Gamma_{\mathrm{grav}} = \frac{-1}{16\pi G} \int d^4x \sqrt{g}\left(R - 2\bar{\lambda}\right) + S_{\mathrm{gf}}, \tag{4}$$

with a gauge fixing term $S_{\mathrm{gf}} = \frac{1}{32\pi\alpha} \int d^4x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu} F_\mu F_\nu$ with $F_\mu = \left(\delta_\mu^\lambda \bar{D}^\kappa - \frac{1}{4}\bar{g}^{\kappa\lambda}\bar{D}_\mu\right) h_{\kappa\lambda}$ and gauge parameter $\alpha = 0$. The Renormalization Group flow for a Litim-type regulator [24], appropriately chosen for fermions [25], in the fermionic sector is driven by two diagrams, cf. Fig. 1.

We use dimensionless couplings and normalize the kinetic term to its canonical form, obtaining

$$\bar{m}_\psi = Z_\psi m_\psi k, \quad \bar{\xi} = Z_\psi \frac{\xi}{k}, \quad \bar{\zeta} = Z_\psi \frac{\zeta}{k},$$
$$G = \frac{g}{k^2}, \quad \bar{\lambda} = \lambda k^2. \tag{5}$$

3. Results: asymptotic safety with heavy and light fermions

In asymptotically safe gravity, a UV completion of the low-energy effective field theory for the metric is provided by an interacting fixed point of the RG flow, generalizing the powerful concept of asymptotic freedom to a quantum-gravitational setting. Previous results indicate that chiral symmetry is not broken spontaneously by asymptotically safe gravity [26–28]. Here, we go one step further and consider explicit breaking terms. We discover two asymptotically safe fixed points. Both provide a viable generalization of the well-known pure-gravity fixed point [2,3,22,29], cf. Table 1. Towards the IR, the RG flow stays within the critical surface of an UV fixed point, if it is fine-tuned to that surface in the UV. This happens automatically, if the flow is set to start at an UV fixed point. On the other hand, UV-relevant directions in the space of couplings are IR-repulsive. This will be decisive for the status of chiral symmetry in quantum gravity. We define the critical exponents as minus the eigenvalues of the stability matrix such that UV-relevant directions have $\theta > 0$,

$$\theta_I = -\mathrm{eig}\left(\frac{\partial \beta_{g_i}}{\partial g_j}\right)_{g_n = g_n^*}, \tag{6}$$

where $g_i = (g, \lambda, m_\psi, \xi, \zeta)_i$.

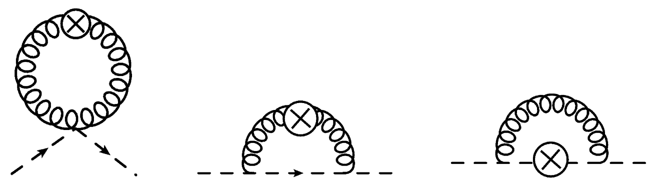

Fig. 1. Three diagrams drive the RG flow in the fermionic couplings. For the two-vertex diagram, the regulator insertion, denoted by a crossed circle, can be found on either of the internal propagators. Curly lines denote the metric propagator and dashed lines the fermions.

As expected due to the preservation of global symmetries in the Wetterich equation, one of the fixed points is governed by an enhanced chiral symmetry and thus enforces a vanishing fermionic mass term at the microscopic level, reproducing the fixed point discussed in detail in [30]. Here, we discover that it features three relevant directions, if we allow non-chiral fluctuations. Further, ξ and ζ align quite well with the irrelevant directions. Thus, both of these couplings are automatically forced to remain zero also in the IR. On the other hand, the mass operator is UV-relevant. For the RG flow towards the IR, our results thus indicate that reaching the chirally symmetric regime requires the tuning of parameters, i.e., chiral symmetry does not act as an infrared attractor. Thus, chiral symmetry is not an IR emergent phenomenon in asymptotic safety, which would have guaranteed the existence of light fermions. However, if chiral symmetry is an exact symmetry of the microscopic theory, it remains unaffected by gravity fluctuations in the asymptotic-safety scenario. Hence, asymptotic safety is *compatible* with the existence of light fermions.

The second fixed point is fully interacting and features a non-vanishing fermion mass, cf. Table 1, generically also resulting in a finite mass in the IR. It again exhibits three relevant directions. The third relevant direction at the non-chiral fixed point is a non-trivial superposition of the fermionic couplings. In particular, this fixed point cannot be discovered in smaller truncations with $\xi = 0$,