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Naturalness in see-saw mechanism and Bogoliubov transformation

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ABSTRACT

We present an alternative perspective on the see-saw mechanism for the neutrino mass, according to which the small neutrino mass is given as a difference of two large masses. This view emerges when an analogue of the Bogoliubov transformation is used to describe Majorana neutrinos in the Lagrangian of the see-saw mechanism, which is analogous to the BCS theory. The Bogoliubov transformation clarifies the natural appearance of Majorana fermions when C is strongly violated by the right-handed neutrino mass term with good CP in the single flavor model. Analyzing typical models with $m_R = 10^4$ to 10^{15} GeV, it is shown that a hitherto unrecognized fine tuning of the order $m_\nu/m_R = 10^{-15}$ to 10^{-26} is required to make the commonly perceived see-saw mechanism work in a natural setting, namely, when none of the dimensionless coupling constants are very small.

1. Introduction

When one discusses the natural appearance of the observed very small neutrino masses [1], one often refers to the see-saw mechanism [2–4] the precise form of which depends on specific models [5]. Those models are characterized by a very large mass scale and thus the natural appearance of the tiny neutrino mass is rather surprising. Naturalness is an esthetic notion and thus subjective, and it should ultimately be determined by experiments. Currently active search for the support of the see-saw mechanism in the form of Majorana neutrinos is going, and we expect that this esthetical issue will be tested soon by experiments.

It may also be appropriate to examine the naturalness of the see-saw mechanism from a different perspective. We attempt to understand the natural appearance of the eigenstates of charge conjugation C, Majorana fermions, using an analogue of the Bogoliubov transformation when C is strongly violated by the right-handed neutrino mass term which has good CP symmetry. We then recognize that the tiny neutrino mass in the see-saw mechanism is given as a difference of two large masses, precise values of which depend on models. This suggests a view different from the conventional one, motivating us to ask whether the see-saw mechanism is "natural" in the sense emphasized, for example, in [6,7]. We show that a hitherto unrecognized fine tuning of the order m_ν/m_R is required to make the see-saw mechanism work in a natural setting.

We first recapitulate the basic properties of Majorana fermions, namely, charge conjugation and parity. The Majorana fermions are defined by the condition

$$\psi(x) = C\bar{\psi}^T(x) = \psi^c(x),$$

where $C = i\gamma^2\gamma^0$ stands for the charge conjugation matrix [8]; the quantity $C\bar{\psi}^T(x)$ is directly evaluated for a given $\psi(x)$ while $\psi^c(x)$ is evaluated by a unitary charge conjugation operator, and the agreement of these two expressions provides an important consistency check in our analysis (for example, of eq. (26) below). We start with a generic neutral Dirac fermion, which is denoted by $\nu(x)$ for later convenience, and define the combinations

$$\psi_\pm(x) = \frac{1}{\sqrt{2}}[\nu(x) \pm \nu^c(x)],$$

which satisfy

$$\psi^c_\pm(x) = \pm\psi_\pm(x),$$

showing that $\psi_+(x)$ and $\psi_-(x)$ are Majorana fields. We treat the fermion with $\psi^c_-(x) = -\psi_-(x)$ also as a Majorana fermion.

It is well-known [8,9] that, in theories where the fermion number is conserved, discrete symmetries such as parity can generally be defined with an arbitrary phase freedom δ,

$$\nu(x) \to e^{i\delta}\gamma^0\nu(t, -\vec{x}).$$

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The conventional parity $\nu(x) \to \gamma^0 \nu(t, -\vec{x})$ and $\nu^c(x) \to -\gamma^0 \nu^c(t, -\vec{x})$ for the Dirac fermion (in the following called "γ^0-parity") corresponds to $\delta = 0$ and thus satisfying $P^2 = 1$. One can confirm that parity for an *isolated single* Majorana fermion is consistently defined only by "$i\gamma^0$-parity" with $\delta = \pi/2$, i.e. $\nu(x) \to i\gamma^0 \nu(t, -\vec{x})$ and $\nu^c(x) \to i\gamma^0 \nu^c(t, -\vec{x})$, namely by (see Ref. [9])

$$\psi_\pm(x) \to i\gamma^0 \psi_\pm(t, -\vec{x}). \tag{1}$$

This definition is consistent with the reality of $\psi_\pm(x)$ in the Majorana representation, where γ^0 is hermitian but purely imaginary. The phase freedom δ is thus fixed by the Majorana condition and $P^2 = -1$. We are interested in Majorana fermions, therefore we exclusively use this "$i\gamma^0$-parity" in this paper.

2. Model Lagrangian and Bogoliubov transformation

We analyze the hermitian Lorentz invariant quadratic Lagrangian for a single flavor of the neutrino, which is a minimal extension of the Standard Model,

$$\begin{aligned}\mathcal{L} &= \overline{\nu}_L(x) i\gamma^\mu \partial_\mu \nu_L(x) + \overline{n}_R(x) i\gamma^\mu \partial_\mu n_R(x) \\ &\quad - m\overline{\nu}_L(x) n_R(x) - (m_L/2)\nu_L^T(x) C \nu_L(x) \\ &\quad - (m_R/2) n_R^T(x) C n_R(x) + h.c.,\end{aligned} \tag{2}$$

where $n_R(x)$ is a right-handed analogue of $\nu_L(x)$, and m, m_L, and m_R are real parameters. We define a new Dirac-type variable

$$\nu(x) \equiv \nu_L(x) + n_R(x) \tag{3}$$

in terms of which the above Lagrangian is re-written as

$$\begin{aligned}\mathcal{L} &= (1/2)\{\overline{\nu}(x)[i\partial\!\!\!/ - m]\nu(x) + \overline{\nu^c}(x)[i\partial\!\!\!/ - m]\nu^c(x)\} \\ &\quad - (\epsilon_1/4)[\overline{\nu^c}(x)\nu(x) + \overline{\nu}(x)\nu^c(x)] \\ &\quad - (\epsilon_5/4)[\overline{\nu^c}(x)\gamma_5\nu(x) - \overline{\nu}(x)\gamma_5\nu^c(x)],\end{aligned} \tag{4}$$

where $\epsilon_1 = m_R + m_L$ and $\epsilon_5 = m_R - m_L$. The C and P transformation rules for $\nu(x)$ are defined by

$$\nu^c(x) = C\bar{\nu}^T(x), \quad \nu^p(x) = i\gamma^0 \nu(t, -\vec{x}), \tag{5}$$

and thus $\nu(x) \leftrightarrow \nu^c(x)$ under C and $\nu^c(x) \to i\gamma^0 \nu^c(t, -\vec{x})$ under P; CP is given by

$$\nu^{cp}(x) = i\gamma^0 C\bar{\nu}^T(t, -\vec{x}). \tag{6}$$

The above Lagrangian (4) is CP conserving, although C and P ($i\gamma^0$-parity) are separately broken by the last term.

In defining Majorana fermions, the exact meaning of the charge conjugation operation C is crucial. In literature (see, e.g., Ref. [5]), one customarily defines the charge conjugation in the Lagrangian (2) by

$$(\nu_L(x))^c = C\bar{\nu}_L^T(x), \quad (n_R(x))^c = C\bar{n}_R^T(x). \tag{7}$$

We must emphasize that the symbols $(\nu_L(x))^c$ and $(n_R(x))^c$ are not to be understood as "transformation laws" but rather as mnemonics for the quantities on the right-hand side, since a unitary operator to generate those transformations does not exist. This can be clearly seen by the following contradictions. If one assumes the action of the unitary charge conjugation operator, one has $\nu_L(x) = [(1-\gamma_5)/2]\nu_L(x)$ and

$$\begin{aligned}(\nu_L(x))^c &= \mathcal{C}\nu_L(x)\mathcal{C}^\dagger = [(1-\gamma_5)/2]\mathcal{C}\nu_L(x)\mathcal{C}^\dagger \\ &= [(1-\gamma_5)/2]C\bar{\nu}_L^T(x),\end{aligned}$$

which imply $(\nu_L(x))^c = 0$, and similarly for $n_R(x)$. Moreover, the well-known C- and P-violating weak interaction Lagrangian is written as

$$\begin{aligned}\mathcal{L}_W &= (g/\sqrt{2})\bar{e}_L\gamma^\mu W_\mu^{(-)}(x))\nu_L + h.c. \\ &= (g/\sqrt{2})\bar{e}_L\gamma^\mu W_\mu^{(-)}(x))[(1-\gamma_5)/2]\nu_L + h.c.\end{aligned} \tag{8}$$

If one assumes again (7) as transformation laws, the first expression implies that $\mathcal{L}_W$ is invariant under C, while the second expression implies $\mathcal{L}_W \to 0$. CP (or CPT) is the only reliable way to define a chiral antiparticle. More comments on this issue will be given later.

The transformation rules (5) for the Lagrangian (4) are operatorially well defined, and they imply

$$\nu_{L,R}^c(x) = \left(\frac{1 \mp \gamma_5}{2}\right)\nu^c(x) = C\overline{\nu}_{R,L}^T(x), \tag{9}$$

as well as

$$\nu_{L,R}^p(x) = i\gamma^0 \nu_{R,L}(t, -\vec{x}), \tag{10}$$

namely, *doublet representations* of C and P for $\nu_L(x)$ and $n_R(x)$, which are not symmetries of (2) for $m_L \neq m_R$. The CP transformation

$$\nu_{L,R}^{cp}(x) = i\gamma^0 C\overline{\nu}_{L,R}^T(t, -\vec{x}) \tag{11}$$

is an exact symmetry of the original Lagrangian (2). We thus adopt the Lagrangian (4) and the (unitary) C and P transformations (5) as the basis of our analysis, which defines a prototype of the Lagrangian of the see-saw mechanism [2–5] for $m_L \simeq 0$, where the right-handed Majorana-type mass m_R is added to the Dirac fermion with mass m. An analogy of the Lagrangian (4) with the Bardeen–Cooper–Schrieffer (BCS) theory was noted some time ago [10].

To solve (4), we apply an analogue of Bogoliubov transformation, $(\nu, \nu^c) \to (N, N^c)$, defined as

$$\begin{pmatrix} N(x) \\ N^c(x) \end{pmatrix} = \begin{pmatrix} \cos\theta\,\nu(x) - \gamma_5\sin\theta\,\nu^c(x) \\ \cos\theta\,\nu^c(x) + \gamma_5\sin\theta\,\nu(x) \end{pmatrix}, \tag{12}$$

with $\sin 2\theta = (\epsilon_5/2)/\sqrt{m^2 + (\epsilon_5/2)^2}$. We can then show that the anticommutators are preserved, i.e.,

$$\begin{aligned}&\{N(t,\vec{x}), N^c(t,\vec{y})\} = \{\nu(t,\vec{x}), \nu^c(t,\vec{y})\}, \\ &\{N_\alpha(t,\vec{x}), N_\beta(t,\vec{y})\} = \{N_\alpha^c(t,\vec{x}), N_\beta^c(t,\vec{y})\} = 0,\end{aligned} \tag{13}$$

and thus it satisfies the canonicity condition of the Bogoliubov transformation. A transformation analogous to (12) has been successfully used in the analysis of neutron-antineutron oscillations [11].

After the Bogoliubov transformation, which diagonalizes the Lagrangian with $\epsilon_1 = 0$, $\mathcal{L}$ in (4) becomes

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[\overline{N}(x)\,(i\partial\!\!\!/ - M)\,N(x) + \overline{N^c}(x)\,(i\partial\!\!\!/ - M)\,N^c(x)\right] \\ &\quad - \frac{\epsilon_1}{4}[\overline{N^c}(x)N(x) + \overline{N}(x)N^c(x)],\end{aligned} \tag{14}$$

with the mass parameter

$$M \equiv \sqrt{m^2 + (\epsilon_5/2)^2}. \tag{15}$$

This implies that the Bogoliubov transformation maps the original theory to a theory characterized by the new large mass scale M ($\epsilon_5/2$ corresponds to the energy gap). The Bogoliubov transformation maps a linear combination of a Dirac fermion and its charge