Physics Letters B 767 (2017) 278–284

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Hyperscaling violating solutions in generalised EMD theory

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ARTICLE INFO

Article history:
Received 19 December 2016
Received in revised form 30 January 2017
Accepted 2 February 2017
Available online 8 February 2017
Editor: M. Cvetič

Keywords:
Holography
Scaling
Hyperscaling violation
Quantum criticality

ABSTRACT

This short note is devoted to deriving scaling but hyperscaling violating solutions in a generalised Einstein–Maxwell-Dilaton theory with an arbitrary number of scalars and vectors. We obtain analytic solutions in some special case and discuss the physical constraints on the allowed parameter range in order to have a well-defined holographic ground-state solution.

1. Introduction

The application of the holographic duality towards understanding theories that are at finite density and may be in the universality class of strongly coupled systems has been made much progress [1,2]. Of particular interests is quantum criticality, which is crucial for interpreting a wide variety of experiments. A large class of critical points in condensed matter are characterised by two scaling exponents, known as the dynamical critical exponent z and the hyperscaling violation exponent θ.

Such exponents appear in holographic saddle point solutions. Lifshitz scaling solutions have been discussed first in [3] (for a recent review see [4]), while hyperscaling violating solutions were recognised in [5,6].

The duality provides a natural framework to describe those quantum critical systems. The metric in the gravitational dual description takes the form

$$ds^2 = r^{\frac{2\theta}{d}} \left(-\frac{dt^2}{r^{2z}} + \frac{L^2 dr^2 + d\vec{x}^2}{r^2} \right), \tag{1.1}$$

with $d\vec{x}^2 = dx_1^2 + \cdots + dx_d^2$ and d the number of spatial dimensions in the field theory. The scaling geometry possesses the property

$$r \to \lambda r, \quad t \to \lambda^z t, \quad x_i \to \lambda x_i, \quad ds^2 \to \lambda^{\frac{2\theta}{d}} ds^2, \tag{1.2}$$

where λ is a dimensionless constant. Therefore, z characterises the deviation from the Lorentz invariant and θ characterises the deviation from the scale invariant limit. Aspects of hyperscaling violating geometry and its realisation in various gravity models have been widely discussed in the literature, see for example [5–11] and references therein. Those geometries with hyperscaling violation are usually considered as the infrared (IR) limit of some kind of bulk solutions that asymptotically approach AdS in the boundary. Due to the presence of nontrivial scaling exponents, there are novel behaviours relative to the AdS counterpart, see for example [12–27]. In particular, it was found that the recent pnictide data [28] can be well described by using holographic DBI magnetoresistance at quantum criticality with hyperscaling violation [29].

It is well known that hyperscaling violating solutions can be generated in the Einstein–Maxwell-Dilaton (EMD) theory where gravity couples to one real neutral scalar and one U(1) gauge field [5,6]. We would like to generalise the simple EMD theory to involve an arbitrary number of scalars and vectors. We will consider a bottom-up theory where the theory parameters can be turned continuously at the level of effective holographic theory. Our motivation are two folds. Firstly, such kind of theory is common by consistent truncation of various supergravity theories in higher dimensions [29–34]. We would like to describe possible geometries in those general setups. Such theory would then be either embedded in string theory/supergravity, or asymptotic to AdS. On the other hand, there have been recently a number of holographic models using multiple vectors and scalars, as they typically lead to

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http://dx.doi.org/10.1016/j.physletb.2017.02.004

richer physics [35–38]. For example, multiple U(1) gauge fields in the bulk will source multiple conserved currents in the dual field theory. Conductivities in such case have been discussed in [39,40], although their physical interpretations are not yet very clear.

We discuss the conditions for the existence of purely scaling geometry in our theory. It turns out that the existence of scaling solutions in general imposes non-trivial constraints on theory parameters. Then we use the established formulae to the special case where the scalars take the standard kinetic term. We can find exact black brane solutions with arbitrary values of hyperscaling violation exponent θ and dynamical exponent z.

The rest of the paper is organised as follows. In section 2 we introduce the gravity theory and derive the equations of motion. Section 3 is devoted to discussing the hyperscaling violating geometry. We give the conditions for the existence of such scaling solutions, which in general can not be solved analytically due to the scalar metric in front of the kinetic terms of scalars. Section 4 presents a set of exact solutions at extremal case as well as finite temperature case. The constraints on the parameter range of (θ, z) are discussed in more details. An example from a top-down setup by using toroidal compactifications is given. We conclude in section 5.

2. The general theory and equations of motion

We consider an effective gravitational theory that involves an arbitrary number of scalars and vector fields at the two-derivative level. The action reads

$$S=\int d^{d+2}x\sqrt{-g}\left[\mathcal{R}-\frac{1}{2}\sum_{i,j=1}^{M}\mathcal{G}_{ij}(\phi)\nabla_\mu\phi_i\nabla^\mu\phi_j+V(\phi)\right.$$
$$\left.-\frac{1}{4}\sum_{I=1}^{N}Z_I(\phi)F_I^2\right], \tag{2.1}$$

which contains M scalars ϕ_i and N massless vectors A_I. We also generalise it a bit by allowing a non-trivial symmetric metric $\mathcal{G}_{ij}(\phi)$ for the scalars.

From the action (2.1) we derive the equations of motion for the scalar ϕ_i

$$\nabla_\mu\left(\sum_{j=1}^{M}\mathcal{G}_{ij}(\phi)\nabla^\mu\phi_j\right)-\frac{1}{2}\sum_{j,k=1}^{M}\frac{\partial\mathcal{G}_{jk}(\phi)}{\partial\phi_i}\nabla_\mu\phi_j\nabla^\mu\phi_k+\frac{\partial V(\phi)}{\partial\phi_i}$$
$$-\frac{1}{4}\sum_{I=1}^{N}\frac{\partial Z_I(\phi)}{\partial\phi_i}F_I^2=0, \tag{2.2}$$

and vector A_I

$$\nabla_\mu(Z_I(\phi)F_I^{\mu\nu})=0, \tag{2.3}$$

with $i, j, k = 1, \dots, M$ and $I = 1, \dots, N$. The equations of motion for the metric $g_{\mu\nu}$ are given by

$$\mathcal{R}_{\mu\nu}-\frac{1}{2}\mathcal{R}g_{\mu\nu}=\frac{1}{2}\sum_{i,j=1}^{M}\mathcal{G}_{ij}(\phi)\left(\nabla_\mu\phi_i\nabla_\nu\phi_j-\frac{1}{2}g_{\mu\nu}\nabla_\rho\phi_i\nabla^\rho\phi_j\right)$$
$$+\frac{1}{2}g_{\mu\nu}V(\phi)$$
$$+\frac{1}{2}\sum_{I=1}^{N}Z_I(\phi)\left(F_{I\mu\rho}F_{I\nu}{}^{\rho}-\frac{1}{4}g_{\mu\nu}F_I^2\right). \tag{2.4}$$

We are interested in the hyperscaling violating solution in the generalised EMD theory. We further simplify the discussion by specialising to the diagonal scalar metric case, i.e., only turn on the diagonal metric $\mathcal{G}_{ii}(\phi)$. We approximate the scalar couplings have exponential asymptotics as in supergravity,

$$\mathcal{G}_{ii}\sim e^{\vec{\tau}_i\cdot\vec{\phi}},\quad V\sim V_0\,e^{-\vec{\delta}\cdot\vec{\phi}},\quad Z_I\sim e^{\vec{\gamma}_I\cdot\vec{\phi}},$$

with V_0 a positive constant. Here we have used a vector notation for M scalars with $\vec{\phi}=(\phi_1,\phi_2,\cdots,\phi_M)$. So the theory we are considering depends on $(M+N+1)$ M-vectors $\vec{\tau}_i$, $\vec{\gamma}_I$ and $\vec{\delta}$. Those vectors will be related to the scaling exponents of the solutions, i.e., z and θ.

In this note we focus on the case with two spatial boundary dimensions ($d = 2$) for simplicity, but our discussion can be generalised to higher dimensions straightforwardly. For the homogeneous and isotropic case the bulk metric as well as matter part takes the generic form,

$$ds^2=-D(r)dt^2+B(r)dr^2+C(r)(dx_1^2+dx_2^2),$$
$$\phi_i=\phi_i(r),\qquad A=A_{It}(r)\,dt. \tag{2.5}$$

Substituting the ansatz into the equations of motion (2.2), (2.3) and (2.4), one obtains the concrete equations of motion for each field

$$\frac{1}{\sqrt{BDC}}\left(\sqrt{\frac{D}{B}}Ce^{\vec{\tau}_i\cdot\vec{\phi}}\phi_i'\right)'-\frac{1}{2B}\sum_{j=1}^{M}e^{\vec{\tau}_j\cdot\vec{\phi}}\tau_{ji}\phi_j'^2$$
$$+\frac{1}{2BD}\sum_{I=1}^{N}e^{\vec{\gamma}_I\cdot\vec{\phi}}\gamma_{Ii}A_{It}'^2-V_0\delta_ie^{-\vec{\delta}\cdot\vec{\phi}}=0, \tag{2.6}$$

$$\left(e^{\vec{\gamma}_I\cdot\vec{\phi}}\frac{C}{\sqrt{BD}}A_{It}'\right)'=0, \tag{2.7}$$

$$\frac{2D''}{D}-\frac{2C''}{C}-\left(\frac{B'}{B}-\frac{C'}{C}+\frac{D'}{D}\right)\frac{D'}{D}+\frac{B'C'}{BC}$$
$$-\frac{2}{D}\sum_{I=1}^{N}e^{\vec{\gamma}_I\cdot\vec{\phi}}A_{It}'^2=0, \tag{2.8}$$

$$\frac{2C''}{C}-\left(\frac{B'}{B}+\frac{C'}{C}+\frac{D'}{D}\right)\frac{C'}{C}+\sum_{i=1}^{M}e^{\vec{\tau}_i\cdot\vec{\phi}}\phi_i'^2=0, \tag{2.9}$$

$$\frac{D'C'}{DC}+\frac{1}{2}\frac{C'^2}{C^2}-\frac{1}{2}\sum_{i=1}^{M}e^{\vec{\tau}_i\cdot\vec{\phi}}\phi_i'^2+\frac{1}{2D}\sum_{I=1}^{N}e^{\vec{\gamma}_I\cdot\vec{\phi}}A_{It}'^2$$
$$-BV_0e^{-\vec{\delta}\cdot\vec{\phi}}=0. \tag{2.10}$$

Here we have used primes to denote radial derivatives. τ_{ji} and γ_{Ii} denote the i-th component of the vectors $\vec{\tau}_j$ and $\vec{\gamma}_I$, respectively.

3. General scaling solutions

We are interested in the hyperscaling violation geometry with the following scaling ansatz

$$ds^2=r^\theta\left[-\frac{dt^2}{r^{2z}}+\frac{L^2\,dr^2+d\vec{x}^2}{r^2}\right],\quad \vec{\phi}=\vec{\kappa}\,\log r,\quad A_{It}=A_{It}(r), \tag{3.1}$$

where $\vec{\kappa}$ is a constant M-vector.

Substituting the above ansatz into (2.7), we find

$$A_{It}''-\frac{1-z-\vec{\gamma}\cdot\vec{\kappa}}{r}A_{It}'=0,\quad I=1,\dots,N \tag{3.2}$$

from which we can determine A_{It}:

$$A_{tI}(r)=\mu_I+Q_I\,r^{2-z-\vec{\gamma}_I\cdot\vec{\kappa}},\quad \vec{\gamma}_I\cdot\vec{\kappa}\neq 2-z. \tag{3.3}$$