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Geometric Monte Carlo and black Janus geometries

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ABSTRACT

We describe an application of the Monte Carlo method to the Janus deformation of the black brane background. We present numerical results for three and five dimensional black Janus geometries with planar and spherical interfaces. In particular, we argue that the 5D geometry with a spherical interface has an application in understanding the finite temperature bag-like QCD model via the AdS/CFT correspondence. The accuracy and convergence of the algorithm are evaluated with respect to the grid spacing. The systematic errors of the method are determined using an exact solution of 3D black Janus. This numerical approach for solving linear problems is unaffected initial guess of a trial solution and can handle an arbitrary geometry under various boundary conditions in the presence of source fields.

1. Introduction

In this note, we shall consider various black Janus geometries numerically in three and five dimensional spaces. Janus geometries are dual to interface (conformal) field theories [1,2],[1] which are well-controlled deformations of the AdS/CFT correspondence [4]. A black Janus geometry is dual to the finite temperature version of the corresponding interface (conformal) field theory. While an exact solution for the 3D black Janus geometry is available [5,6], we shall numerically reconsider it for a geometric interpretation of Monte Carlo (MC) method. In five dimensions, we shall consider two cases: one with a planar interface and the other with a spherical interface. In the latter, the boundary value of the scalar field, whose exponential is corresponding to the Yang–Mills (YM) coupling squared divided by 4π, has a smaller value inside the sphere than outside. Its dual field theory, whose finite-temperature counterpart shall be considered below, resembles the MIT bag model in QCD [7].

As for the numerical analysis, we shall use the standard MC method [8] (see [9] for a general review) to solve the scalar field equations in the black brane background which are elliptic partial differential equations (PDEs). The choice of the MC method is conceptually motivated by the following considerations. In the MC method for a PDE, an estimate to the solution at each site is evaluated with an average of samples of boundary values by generating a sufficiently large number of random walks each of which starts from the original site and ends at one of the boundary sites. The direction of the movement in each step of the random walk will be chosen randomly with respect to the probabilities determined by the associated PDE. As we will see, the probabilities at each site fully reflect the underlying geometry and hence these random walks may be regarded as processes exploring geometric landscapes rather efficiently. This feature coins the name of "geometric MC method" and is the reason why this MC method provides an interesting framework for the numerical study of gravity problems. Adding to this, the independence between random walks allows high performance parallel computing to speed up the convergence of the MC simulations. Along with improved computational capabilities, this MC method has some advantages compared to other numerical methods. One can use our MC method on gravitational problems with arbitrary geometries with various boundary conditions. This approach for solving linearized equations does not require any trial configuration.

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[1] For a recent discussion of Janus systems, see [3], where one may find a rather comprehensive list of references on the subject.

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This paper is organized as follows. In sec. 2, we describe a theoretical background for the linearized black Janus. Numerical details of our geometric MC method are discussed in sec. 3. In sec. 4, we present our numerical analysis of 5D black Janus with planar and spherical interfaces, followed by concluding remarks in sec. 5.

2. Black Janus deformations to the leading order

In this note, we shall consider the Einstein-scalar system with a negative cosmological constant described by the action

$$I = -\frac{1}{16\pi G}\int d^d x\sqrt{g}\left[R - g^{ab}\partial_a\phi\partial_b\phi + \frac{(d-1)(d-2)}{\ell^2}\right] \quad (2.1)$$

where ℓ is the AdS radius scale which we shall set to be unity for our numerical study below. For $d=3$ and 5, this system can be consistently embedded into the type IIB supergravity and, hence, via the AdS/CFT correspondence, microscopic understanding of dual CFT$_{d-1}$ can be given [1]. In particular, in five dimensions, the dual CFT$_4$ is identified with the well known $\mathcal{N}=4$ SU(N) super-Yang–Mills (SYM) theory [4]. The scalar field originated from the dilaton of the type IIB supergravity is dual to the Lagrange-density operator of the SYM theory, whose boundary value corresponds to the logarithm of the YM coupling squared divided by 4π in the field-theory side.

The finite-temperature black brane background is given by

$$ds^2 = \frac{1}{z^2}\left[(1-z^{d-1})d\tau^2 + \frac{dz^2}{1-z^{d-1}} + dx_1^2 + dx_2^2 + \cdots + dx_{d-2}^2\right] \quad (2.2)$$

with a trivial scalar field $\phi = \phi_0$. By requiring the regularity of geometry around $z=1$ in (τ, z) space, the period of τ-direction angle variable can be identified, whose inverse is the Gibbons–Hawking temperature of the boundary system, $T = (d-1)/(4\pi\ell)$. This black brane background is dual to the finite-temperature version of CFT$_{d-1}$ on $\mathbb{R}\times\mathbb{R}^{d-2}$. The temperature may be scaled to other values by appropriate scaling transformations but it plays a role of unique reference scale in this pure black brane background.

In this work, we shall consider various Janus deformations of the above black brane background. The Janus deformation in the bulk involves a scalar field whose boundary values jump from one to another across an interface. The dual boundary system is described by an interface CFT where its original CFT is deformed by an exactly marginal operator which is dual to the bulk scalar field. From the viewpoint of the boundary, its coupling jumps across the interface from one value to another whose detailed identification is subject to the standard dictionary of the AdS/CFT correspondence. In $d=3$, to the leading order of the deformation parameter, the profile of the scalar field is governed by

$$(1-X^2)\partial_X\left[(1-X^2)\partial_X\phi\right] + 4p(1-p)\partial_p^2\phi - 4p\partial_p\phi = 0 \quad (2.3)$$

where we introduce new coordinates $(X,\ p)$ by $X = \tanh x_1$ and $p = z^2$. Here we shall consider the case of a single interface which is located at $x_1 = X = 0$. Since the constant solution $\phi = \phi_0$ can be added freely, the Janus boundary condition can be given as

$$\phi(X,0) = \gamma\ \mathrm{sign}(X) \quad (2.4)$$

where γ is our deformation parameter referred to as an 'interface coefficient'. Of course one may consider the case of multiple interfaces [6] but here we would like to focus on the case of a single interface. Since the leading order is linear, we shall omit the γ dependence for the simplicity of our presentation. Of course the validity of our approximation requires $\gamma \ll 1$ and our numerical result for the scalar profile should be understood with an extra multiplication factor of γ throughout this note. The boundary condition at $X=\pm 1$ then becomes $\phi(\pm 1, p) = \pm 1$. On the horizon side, one may impose the 'Neumann boundary condition' $\left[\sqrt{1-p}\,\partial_p\phi(X,p)\right]\big|_{p=1} = 0$. Note that the $(\tau,\ p)$ plane of the black brane geometry has a shape of an infinite sized disk whose center is located at $p=1$. Near this center $1-p\sim 0$, the distance from the center is approximately given by $s\sim\sqrt{1-p}$. Then the above boundary condition follows from the Neumann boundary condition $\partial_s\phi\big|_{s=0} = 0$ with respect to the distance s, which ensures the smoothness of our scalar profile at $s=0$. Below we shall replace this boundary condition by a smoothness condition of the scalar field at $p=1$ which basically allows us to Taylor-expand $\phi(X,p)$ around $p=1$ to some orders, whose details will be further specified in our numerical study below. We shall refer to this as a 'free boundary condition'.

Now note that our system possesses a $\mathbb{Z}_2$ symmetry $\phi(X,p) = -\phi(-X,p)$. So the problem can be reduced to solving the differential equation restricted in the region of $X\geq 0$ with the boundary condition at $X=0$ specified by $\phi(0,p)=0$. In this $d=3$ case, an exact solution can be found as [5]

$$\phi = \gamma\frac{X}{\sqrt{X^2 + p(1-X^2)}} \quad (2.5)$$

Even an analytic black Janus solution including the full gravitational back-reaction has been found in [5]. Thus this 3D problem will serve as a nice testing ground for the methods we use for our numerical study below.

Now let us turn to our main theme which is the $d=5$ case. This is relevant to the problem of understanding properties of $\mathcal{N}=4$ SYM theory. Especially in its finite temperature version, it has been argued to be useful in understanding certain aspects of the real-world QCD although its full justification is not that straightforward [10].

Again in the probe limit, the 5D scalar equation is reduced to

$$\left(\partial_{x_1}^2 + \partial_{x_2}^2 + \partial_{x_3}^2\right)\phi + 4p(1-p^2)\partial_p^2\phi - 4(1+p^2)\partial_p\phi = 0 \quad (2.6)$$

where $p = z^2$ as before. We shall first study a Janus deformation involving a single planar interface located at $x_1 = 0$ which has translational symmetries along x_2 and x_3 directions. Thus with $\partial_{x_2}\phi = \partial_{x_3}\phi = 0$, the scalar profile is governed by

$$(1-X^2)\partial_X\left[(1-X^2)\partial_X\phi\right] + 4p(1-p^2)\partial_p^2\phi - 4(1+p^2)\partial_p\phi = 0 \quad (2.7)$$

where we introduce X by $X = \tanh x_1$ as before. For this planar interface, we have the boundary conditions $\phi(0,p) = 0$, $\phi(X,0) = \phi(1,p) = 1$ together with the Neumann boundary condition $\left[\sqrt{1-p}\,\partial_p\phi(X,p)\right]\big|_{p=1} = 0$, which can be replaced by the free boundary condition at $p=1$ as before. We shall solve the equation for the half region of $X\geq 0$ utilizing the underlying $\mathbb{Z}_2$ symmetry. Below we shall pay a particular attention to the horizon profile, $\phi(X,1)$, of our scalar field to see how the horizon is colored by the scalar hair.

Next we would like to consider a bag-like configuration as an application of the Janus deformation of the black brane background. For this, we introduce a boundary radial coordinate r defined by $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ and impose the boundary condition $\phi(r,0) = -\gamma\,\theta(R-r)$ where $\theta(x)$ denoting the Heaviside step function. This boundary condition describes a bag-like model where the YM coupling $g_{YM}^2 = 4\pi\, e^{\phi(r,0)+\phi_0}$ in the region of $r\leq R$ becomes weaker than the one outside the bag. Later we shall argue that hadrons can be realized by a fundamental (QCD) string corresponding to a Wilson line connecting quark to anti-quark in the