Physics Letters B 767 (2017) 386–391

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Configurational entropy of anti-de Sitter black holes

Nelson R.F. Braga [a], Roldão da Rocha [b,*]

[a] Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, RJ 21941-972, Brazil
[b] Centro de Matemática, Computação e Cognição, Universidade Federal do ABC – UFABC, 09210-580, Santo André, Brazil

ARTICLE INFO

Article history:
Received 20 December 2016
Received in revised form 31 January 2017
Accepted 10 February 2017
Available online 21 February 2017
Editor: N. Lambert

Keywords:
Configurational entropy
Gauge-gravity correspondence
AdS-Schwarzschild black holes
Hawking–Page transition

ABSTRACT

Recent studies indicate that the configurational entropy is an useful tool to investigate the stability and (or) the relative dominance of states for diverse physical systems. Recent examples comprise the connection between the variation of this quantity and the relative fraction of light mesons and glueballs observed in hadronic processes. Here we develop a technique for defining a configurational entropy for an AdS-Schwarzschild black hole. The achieved result corroborates consistency with the Hawking–Page phase transition. Namely, the dominance of the black hole configurational entropy will be shown to increase with the temperature. In order to verify the consistency of the new procedure developed here, we also consider the case of black holes in flat space-time. For such a black hole, it is known that evaporation leads to instability. The configurational entropy obtained for the flat space case is thoroughly consistent with the physical expectation. In fact, we show that the smaller the black holes, the more unstable they are. So, the configurational entropy furnishes a reliable measure for stability of black holes.

1. Introduction

A black hole is a particular solution of Einstein equations and has both a temperature related to its surface gravity and entropy, this last one associated with the black hole area. Black holes can radiate and evaporate, due to particle creation and annihilation, quantum fluctuations near the event horizon and also due to tunnelling methods across it. AdS-Schwarzschild black holes can be thermodynamically stable and were shown to manifest a first-order phase transition, due to a negative free energy at high temperatures [1]. The study of black holes in anti-de Sitter (AdS) space was, afterwards, refreshed in the AdS/CFT correspondence, comprising a relationship between string theory in a 5-dimensional AdS space and Yang–Mills theories on the AdS space conformal boundary. Subsequently, the Hawking–Page phase transition was understood in the gauge theory setup as a phase transition, from confinement to deconfinement [2,3].

One of the most striking applications of the informational entropy in physics is to provide a remarkable tool to analyze compact astrophysical objects. In fact, a stability bound for stellar distributions was obtained in the context of the configurational entropy, both in Refs. [4,5] and in Ref. [6] as well. This last result provided a relevant procedure to scrutinize the stability of Bose–Einstein condensates of long-wavelength gravitons in a black hole quantum portrait, in the configurational entropy setup [6]. In both these developments, the critical stability region of stellar configurations have been consistently defined in complementary, distinct, paradigms that match observational data [4,6]. Hence, the critical stellar densities were shown to match the Chandrasekhar stellar critical density, associated with critical points of the informational entropy.

The present day concept of configurational entropy has roots in early studies of informational entropy by Shannon [7], introduced in the context of communication theory. The earliest main idea was to associate different messages to distinct probabilities. When N messages are equally probable, Shannon defined the informational entropy of an unread message as being $\log_2 N = k_S \log N$, with $k_S = 1/\ln 2$ playing a role analogous to the Boltzmann constant. Entropy may be seen as a measure of the information deficit about a system. The informational entropy was re-introduced, as the configurational entropy, in the recent literature by Gleiser and collaborators, in the context of complex systems and also compact astrophysical objects, among other models [4,5,8,9]. It represents a quantitative measure of an entropy of shape, based on successful scrutiny of some physical systems supporting local interactions. The core of the configurational entropy is constituted by

* Corresponding author.
E-mail addresses: braga@if.ufrj.br (N.R.F. Braga), roldao.rocha@ufabc.edu.br (R. da Rocha).

http://dx.doi.org/10.1016/j.physletb.2017.02.031
0370-2693/

the modal fractions in informational entropy, emulating the collective coordinates-to-structure factor ratio in the thermodynamical entropy setup [10].

The configurational entropy, also known as informational entropy,[1] has been recently turned into a promising tool for investigating the stability and/or the dominance of states in some physical models. For example, the configurational entropies of glueball and meson states, as described in the AdS/QCD framework, were recently studied in [10,12], providing prominent applications in the study of hadronic processes. A clear relation between the configurational entropy and the abundance of the states was derived, wherein the states with smaller configurational entropy are more likely found in hadronic processes [10,12].

The purpose of the present paper is to develop an approach for a consistent definition of the configurational entropy for an AdS-Schwarzschild black hole. We want to verify whether the Hawking–Page transition is appropriately represented in the configurational entropy setup. As a by-product, and as a check of consistency, we will also consider a black hole in flat space, for which the evaporation by Hawking radiation leads to the black hole instability. This paper is organized as follows: Sect. 2 is committed to briefly revisit the configurational entropy setup and Sect. 3 is devoted to the construction of a configurational entropy for the AdS-Schwarzschild black hole. In Sect. 4 the flat space case is accomplished and analyzed. Sect. 5 encloses the concluding remarks and outlook.

2. Configurational entropy framework

The configurational entropy is defined from the modal fraction, that is constructed upon of the Fourier transform of the energy density associated with a physical system [4]. The starting point is the informational entropy, originally defined for any system with N discrete modes by the expression $S_c = -\sum_{i=1}^{N} f_i \ln(f_i)$, where the $\{f_i\}$ are probability density functions [7]. For the case of a black hole, the energy density is assumed to be a function of the position $\rho = \rho(\vec{r})$. In what follows, all integrals are defined over the whole space. In a space of n spatial dimensions, the Fourier transform of the energy density,

$$\tilde{\rho}(\vec{\omega}) = (2\pi)^{-n/2}\int \rho(\vec{r})\, e^{i\vec{\omega}\cdot\vec{r}} d^n r \,, \tag{1}$$

can be thought as the continuum limit of the collective coordinates in statistical mechanics, $\rho(\vec{r}) = \sum_{j=1}^{N} \exp\left(-i\vec{\omega}_j\cdot\vec{r}\right)\tilde{\rho}(\vec{\omega}_j)$ [10]. The structure factor, $s_N = \frac{1}{N}\sum_{j=1}^{N}\langle\,|\,\tilde{\rho}(\vec{\omega}_j)\,|^2\rangle$, normalizes the correlation of collective coordinates, as

$$f(\vec{\omega}_N) = \frac{1}{s_N N}\langle\,|\,\tilde{\rho}(\vec{\omega}_N)\,|^2\rangle. \tag{2}$$

The structure factor measures fluctuations in the energy density, providing the system profile towards homogenization. The $N \to \infty$ limit yields the modal fraction to read:

$$f(\vec{\omega}) \equiv \frac{\langle\,|\tilde{\rho}(\vec{\omega})|^2\rangle}{\mathcal{N}}\,, \tag{3}$$

where

$$\mathcal{N} = \int \langle\; |\left(\tilde{\rho}(\vec{\omega})\right)^* \tilde{\rho}(\vec{\omega})|\;\rangle d^n\omega\,. \tag{4}$$

The configurational entropy is, then, defined as [8,9]

$$S_c[f] = -\int f(\vec{\omega}) \ln f(\vec{\omega}) d^n\omega. \tag{5}$$

[1] Ref. [10] also discusses the configurational entropy as a particular case of the conditional relative entropy [11], to better provide a formal study of the integral that defines the configurational entropy.

Critical points of the configurational entropy imply that the system has informational entropy that is critical with respect to the maximal modal fraction $f_{\max}(\vec{\omega})$, corresponding to more dominant states [6,10]. The prototypical d-dimensional distribution is a Gaussian [8], wherein the configurational entropy estimates the information demanded in the reciprocal space to constitute the energy density in the position space. Such a distribution represents an absolute minimum for spatially-localized functions [8]. Dynamical 5-dimensional kinks were also studied in Ref. [13], in this context.

3. AdS-Schwarzschild black holes and information entropy

The thermodynamics of black holes in AdS space have been deeply studied in Ref. [1]. One of the relevant outcomes of this article was the discovery of a phase transition between the black hole space and the thermal AdS space without a black hole. For temperatures above a critical value, the black hole state is dominant, whereas below the critical temperature the thermal AdS space state represents the dominant configuration. Although black holes are at thermal equilibrium with radiation, they are not the preferred state below a certain critical temperature [1]. Subsequently to this investigation, but shortly after the discovery of the AdS/CFT correspondence [2,14,15], the connection between the thermal properties of an AdS-Schwarzschild black hole and those of a dual gauge theory were unveiled in Ref. [3]. The thermal phase transition found in Ref. [1], presently known as Hawking–Page transition, was interpreted in Ref. [3] as a transition between a deconfined phase (corresponding to a high temperature) and a confined phase (low temperature). In order to understand the phase transition, one has to compare the actions of the AdS metric and the AdS-Schwarzschild metric. The phase transition occurs in the regime where the two actions are equal [1].

The anti-de Sitter space is a solution of the vacuum Einstein equation with a negative cosmological constant. The metric of a (thermal) $(n+1)$-dimensional Euclidean AdS space, in global coordinates, can be written as:

$$ds^2 = \left(1+\frac{r^2}{b^2}\right)d\bar{t}^{\,2} + \frac{dr^2}{1+\frac{r^2}{b^2}} + r^2 d\Omega^2_{(n-1)}\,. \tag{6}$$

The boundary of the space, at $r\to\infty$, is the product of a $(n-1)$-sphere $\boldsymbol{S}^{n-1}$ and the circumference S^1, associated with the temporal variable that has a period $\bar{\beta}$, where the equivalence $\bar{t}\sim\bar{t}+\bar{\beta}$ defines the time circle S^1.

On the other hand, an AdS-Schwarzschild black hole metric, with the same asymptotic (large r) boundary reads

$$ds^2 = \left(1+\frac{r^2}{b^2}-\frac{w_n M}{r^{n-2}}\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{b^2}-\frac{w_n M}{r^{n-2}}} + r^2 d\Omega^2\,, \tag{7}$$

according to the notation in Ref. [3], where the constant w_n is related to the black hole mass M and to the $n+1$-dimensional Newton constant G_N

$$w_n = \frac{16\pi G_N}{(n-1)\mathrm{Vol}(S^{n-1})}\,, \tag{8}$$

where $\mathrm{Vol}(S^{n-1}) = 2\pi^{n/2}/\,\Gamma(n/2)$ is the volume of $\boldsymbol{S}^{n-1}$.

The location of the black hole horizon is the radial position where the time component of the metric vanishes and the radial component of the metric becomes singular. Hence, the black hole outer region corresponds to $r > r_h$, with r_h calculated as the largest root of the algebraic equation

$$1+\frac{r^2}{b^2}-\frac{w_n M}{r^{n-2}} = 0\,. \tag{9}$$