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Response function of the large-scale structure of the universe to the small scale inhomogeneities

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A B S T R A C T

In order to infer the impact of the small-scale physics to the large-scale properties of the universe, we use a series of cosmological *N*-body simulations of self-gravitating matter inhomogeneities to measure, for the first time, the response function of such a system defined as a functional derivative of the nonlinear power spectrum with respect to its linear counterpart. Its measured shape and amplitude are found to be in good agreement with perturbation theory predictions except for the coupling from small to large-scale perturbations. The latter is found to be significantly damped, following a Lorentzian form. These results shed light on validity regime of perturbation theory calculations giving a useful guideline for regularization of small scale effects in analytical modeling. Most importantly our result indicates that the statistical properties of the large-scale structure of the universe are remarkably insensitive to the details of the small-scale physics, astrophysical or gravitational, paving the way for the derivation of robust estimates of theoretical uncertainties on the determination of cosmological parameters from large-scale survey observations.

1. Introduction

The cosmic energy fluctuations on large scales provide rich probes of the early universe physics, the mass of neutrinos or the nature of dark energy. Wide-field galaxy surveys are therefore widely considered for unveiling the details of the universe [1]. Among them are the DES,[1] LSST[2] and Euclid[3] now under development. Such measurements rely however largely on our understanding of the statistical properties of the cosmic fluctuations. The great success of the latest cosmic microwave background observations in establishing the standard picture of our Universe largely owed to the fact that the measured temperature fluctuations are in the linear regime and thus can accurately be predicted using linear theory [2,3]. Likewise, we expect that the late-time fluctuations on large scales are in a mildly nonlinear stage, and there are robust ways to predict them precisely beyond linear-theory calculations.

Established probes such as the baryon acoustic oscillations (BAOs; e.g. [4,5]) that give us a robust standard ruler useful for dark energy studies, or the redshift-space distortions (e.g., [6]) as an additional clue to discriminate gravity theories [7], are among those that we expect in a mildly nonlinear regime. Alternatively, we can access cosmic fluctuations on similar and somewhat smaller scales with weak-lensing measurements (see [8] for a recent review). Such scientific programs can only be achieved if related observables can be accurately predicted either from numerical simulations or analytically for any given cosmological model. In particular it is important that such observables are shielded from the details of astrophysics at galactic or sub-galactic scales.[4]

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[1] https://www.darkenergysurvey.org/.
[2] http://www.lsst.org/lsst/.
[3] http://www.euclid-ec.org.

[4] For instance in [9–11] baryonic effects are shown to be confined within the cluster scale, and they contribute to the matter power spectrum at most $\sim 10\%$ at $k = 1\,h\,\mathrm{Mpc}^{-1}$ and drops rapidly toward larger scales.

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One way to reformulate this question is to quantify the impact of small-scale structures on the growth of large scale modes. Perturbation theory (PT) is a powerful framework to predict the growth of structure. Assuming that the system is described by self-gravitating pressure-less fluids, it provides the first-principle approach to the nonlinear growth (see [12] for a review). Its importance has been heightened after the detection of BAOs in the clustering of galaxies, making precise predictions of nonlinearities crucially important.

PT calculations show precisely that mode couplings between different scales are unavoidable. We propose here to quantify these couplings with a two-variable response function,[5] defined as the linear response of the *nonlinear* power spectrum at wave mode k with respect to the *linear* counterpart at wave mode q[6]:

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}. \tag{1}$$

In the context of PT calculations, Refs. [14,15] showed progressive broadening of the response function with increasing PT order, pointing to the need of regularization of the small-scale contribution.

If the broadness of the response function at late times is true, physics at very small scale can influence significantly the matter distribution on large scales, where the acoustic feature is prominent.[7] It also questions the reliability of simulations, which can follow the evolution of Fourier modes only in a finite dynamical range. We here discuss the response function at the non-perturbative level utilizing cosmological N-body simulations.

2. Methodology

We here describe our method to measure the response function from simulations. We prepare two initial conditions with small modulations in the linear spectrum over a finite interval of wave mode q, evolve them to a late time, and take the difference of the nonlinear spectra measured from the two. That is

$$\hat{K}_{i,j}P^{\rm lin}_j \equiv \frac{P^{\rm nl}_i[P^{\rm lin}_{+,j}] - P^{\rm nl}_i[P^{\rm lin}_{-,j}]}{\Delta\ln P^{\rm lin}\Delta\ln q}, \tag{2}$$

where the two perturbed linear spectra are given by

$$\ln\left[\frac{P^{\rm lin}_{\pm,j}(q)}{P^{\rm lin}(q)}\right] = \begin{cases} \pm\frac{1}{2}\Delta\ln P^{\rm lin} & \text{if } q\in[q_j,q_{j+1}),\\ 0 & \text{otherwise.}\end{cases} \tag{3}$$

In the above, the index i (j) runs over the wave-mode bins for the nonlinear (linear) spectrum, and we choose log-equal binning, $\ln q_{j+1} - \ln q_j = \ln k_{i+1} - \ln k_i = \Delta\ln q$. It is straightforward to show that the estimator $\hat{K}$ approaches to the response function K defined in Eq. (1), when $\Delta\ln q$ and $\Delta\ln P^{\rm lin}$ are small. The definition (1) is advantageous in that it allows the measurement in this way at the fully nonlinear level.[8] Note that a similar function was first discussed numerically in Ref. [18] in the context of local transformations of the density field.

We adopt a flat-ΛCDM cosmology consistent with the five-year WMAP result [19] with parameters $(\Omega_{\rm m}, \Omega_{\rm b}/\Omega_{\rm m}, h, A_{\rm s}, n_{\rm s}) = (0.279, 0.165, 0.701, 2.49\times10^{-9}, 0.96)$, which are the current matter density parameter, baryon fraction, the Hubble constant in units of 100 km/s/Mpc, the scalar amplitude normalized at $k_0 = 0.002\,{\rm Mpc}^{-1}$ and its power index, respectively. We also consider different cosmologies to check the generality of the result. Since we can check the dependence of the response function on the overall amplitude of the power spectrum by looking at the results at different redshifts, we here focus on the variety in only the shape of the spectrum. As a representative of the parameters that control the shape, we consider the spectral tilt $n_{\rm s}$. We run simulations for two additional models, one with a higher (1.21; `high_ns`) and the other with a lower (0.71; `low_ns`) value of $n_{\rm s}$. Although the parameter $n_{\rm s}$ has been constrained very tightly from observations of the cosmic microwave background (with only $\sim 1\%$ uncertainty), we choose to give it a rather large (±0.25) variation to cover a wider class of models with different linear power spectra. The amplitude parameter $A_{\rm s}$ for these models is determined such that the rms linear fluctuation at $8\,h^{-1}$ Mpc is kept unchanged. The matter transfer function is computed for these models using the `CAMB` code [20] with the high-precision mode of the calculation in the transfer function (`transfer_high_precision` is set to be true and `accuracy_boost=2`) up to $k_{\rm max} = 100\,h\,{\rm Mpc}^{-1}$. We confirm that the result is well converged by testing more strict values in the parameter file.

Table 1
Simulation parameters. Box size (box), softening scale (soft) and mass of the particles (mass) are respectively given in unit of h^{-1} Mpc, h^{-1} kpc and $10^{10}h^{-1}M_\odot$. The number of q-bins is shown in the "bins" column, for each of which we run two simulations with positive and negative perturbations in the linear spectrum. The "runs" column shows the number of independent initial random phases over which we repeat the same analysis. The total number of simulations are shown in the "total" column.

name	box	particles	$z_{\rm start}$	soft	mass	bins	runs	total
`L9-N10`	512	1024^3	63	25	0.97	5	1	10
`L9-N9`	512	512^3	31	50	7.74	15	4	120
`L9-N8`	512	256^3	15	100	61.95	13	4	104
`L10-N9`	1024	512^3	15	100	61.95	15	1	30
`high_ns`	512	512^3	31	50	7.74	5	4	40
`low_ns`	512	512^3	31	50	7.74	5	4	40

We run four sets of simulations for the fiducial model with different volume and number of particles as listed in Table 1. Covering different wave number intervals, these simulations allow us to examine the convergence of the measured response function. The initial conditions are created using a code developed in [21,22] based on the second-order Lagrangian PT (e.g., [23,24]). The initial redshifts of the simulations are determined as follows. A lower starting redshift can induce transient effects associated with higher-order decaying modes. On the other hand, as increasing the initial redshift, the randomly generated particle position generally gets closer to the pre-initial grid, and this can lead to discreteness noise in the force calculation. To minimize the sum of these two systematic effects, we set the initial redshift such that the rms displacement is roughly 20% of the inter-particle spacing, and thus it depends on the resolution as shown in Table 1. We evolve the matter distribution using a Tree-PM code `Gadget2` [25]. We finally measure the power spectrum by fast Fourier transform of the Cloud-in-Cell (CIC) density estimates on 1024^3 mesh with the CIC kernel deconvolved in Fourier space.

For each set of simulations, we prepare multiple initial conditions with linear spectra perturbed by $\pm1\%$ over $q_j \le q < q_{j+1}$. The amplitude of perturbation should be sufficiently small such that the correction from the higher-order derivative ($\delta^2 P^{\rm nl}/\delta P^{\rm lin}\delta P^{\rm lin}$) does not contaminate the result. We tested different amplitudes ($\pm3\%$ and $\pm5\%$), and confirmed that the result is almost unchanged. We set the bin width as $\Delta\ln q = \ln(\sqrt{2})$ and each simulation set covers different wavenumber range corresponding to

[5] This concept was recently utilized in Ref. [13] to compute the difference of the nonlinear power spectrum for slightly different cosmological models.
[6] The normalization is such that K contributes to the change in $P^{\rm nl}$ with uniform weights per decade.
[7] Notice, however, that the feature can also be affected by galaxy bias [16,17].
[8] This is contrasted to the function F_n appearing in PT for the n-th order coupling.

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