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Mixed heavy–light matching in the Universal One-Loop Effective Action

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ABSTRACT

Recently, a general result for evaluating the path integral at one loop was obtained in the form of the Universal One-Loop Effective Action. It may be used to derive effective field theory operators of dimensions up to six, by evaluating the traces of matrices in this expression, with the mass dependence encapsulated in the universal coefficients. Here we show that it can account for loops of mixed heavy–light particles in the matching procedure. Our prescription for computing these mixed contributions to the Wilson coefficients is conceptually simple. Moreover it has the advantage of maintaining the universal structure of the effective action, which we illustrate using the example of integrating out a heavy electroweak triplet scalar coupling to a light Higgs doublet. Finally we also identify new structures that were previously neglected in the universal results.

1. Introduction

Matching from an ultraviolet (UV) theory to a low-energy effective field theory (EFT) can be performed using either Feynman diagrams or functional methods. For the latter approach, Gaillard [1] and Cheyette [2] introduced a manifestly gauge-covariant method of performing the calculation, using a covariant derivative expansion (CDE). This elegant method simplifies evaluating the quadratic term of the heavy fields in the path integral to obtain the low-energy EFT, and was revived recently by Henning, Lu and Murayama (HLM) [3]. In particular, HLM pointed out that under the assumption of degenerate particle masses they could evaluate the momentum dependence of the coefficients that factored out of the trace over the operator matrix structure, without specifying the specific UV model. In Ref. [4] some of us showed that this universality property can be extended without any assumptions on the mass spectrum, to obtain a universal result for the one-loop effective action for operators up to dimension six. There the loop integrals have been computed for a general mass spectrum once and for all. This Universal One-Loop Effective Action (UOLEA) is a general expression that may then be applied in any context where a one-loop path integral needs to be computed, as for example in matching new physics models to the Standard Model (SM) EFT.[1]

Functional methods require the term quadratic in the heavy fields to be integrated out, corresponding to loops of heavy fields with light particle external legs in the Feynman diagram approach. In addition to these heavy–heavy loops, there could also be mixed heavy–light contributions to matching. These are typically calculated using Feynman diagrams [12,15–17] but can also be accounted for in the functional approach [18–20]. The purpose of this paper is to show how they can be computed in the UOLEA.

Compared to previous functional methods [18–20], our prescription for treating mixed heavy–light contributions is relatively simple and transparent: in addition to the usual expansion of the heavy fields around their classical solution, we also separate the light fields into classical and quantum parts, and extend the quadratic term to also include quantum fluctuations of the light fields. This essentially amounts to computing the 1PI effective action for the full theory, from which the Wilsonian effective Lagrangian, namely the low-energy EFT, can be extracted. Similarly to the heavy–heavy case, the general structure and universal coefficients of the UOLEA combine to yield the EFT Wilson coefficients

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[1] For recent matching calculations see for example Refs. [3–12]. The SM EFT is reviewed in Refs. [13,14].

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after evaluating the matrix traces. But in this extended case, the universal coefficients contain parts that are in the full 1PI effective action but not in the EFT, diagrammatically corresponding to tree-generated operator insertions in EFT loops. These must be subtracted by a well-defined procedure, which we describe. Our prescription has the advantage of maintaining the universal structure of the UOLEA so that in principle, one needs not apply the CDE starting from the beginning for every model.

We also find that in certain cases, for example when including vector gauge boson contributions, the matrix structure may contain an extra covariant derivative part that is not taken into account in the pre-evaluated form of the UOLEA; Refs. [3,4] assume no such additional structure in its derivation. These new contributions then have to be computed separately for each specific case using the CDE method to evaluate the path integral from the beginning. However, it is possible in principle to do the calculation in a model-independent way, once and for all, which would extend the UOLEA to include such structures. Such an extension will be addressed in future work [21].

In the next Section we give a brief introduction to the CDE method and the UOLEA. In Section 3 we outline the procedure for including mixed heavy–light contributions to dimension-6 operators with the UOLEA. As an example, in Section 4 we demonstrate how to obtain heavy–light contributions to matching a heavy electroweak triplet scalar model to the SM EFT, and discuss the extension needed to incorporate gauge coupling-dependent contributions. Finally we conclude in Section 5. Some useful formulae are collected in the Appendix.

2. The Universal One-Loop Effective Action

We begin by describing the Gaillard–Cheyette Covariant Derivative Expansion (CDE) method [1,2] for evaluating the path integral.[2] The UV Lagrangian for a model composed of light and heavy fields, that we collectively denote as the multiplets ϕ and Φ respectively, can be written as

$$\mathcal{L}_{\text{UV}}[\phi,\Phi]\supset\mathcal{L}[\phi]+\Phi\cdot F[\phi]+\frac{1}{2}\Phi(P^2-\mathcal{M}^2-U'[\phi])\Phi+\mathcal{O}(\Phi^3)\,, \tag{2.1}$$

where $\mathcal{L}[\phi]$ is the light field part of the Lagrangian and the gauge-covariant derivative D_μ is written as $P_\mu\equiv iD_\mu$. $\mathcal{M}$ is a diagonal mass matrix. Eq. (2.1) is written for a real scalar Φ; in general the exact form depends on the nature of Φ. The terms involving light fields coupling linearly and quadratically to Φ are represented by the matrices $F[\phi]$ and $U'[\phi]$ respectively.

Beginning from an action $S[\phi,\Phi]$, we can expand around the minimum and evaluate the path integral over Φ. For example in the case of real scalar fields the effective action can be written as

$$\begin{aligned}
e^{iS_{\text{eff}}[\phi]}&=\int[D\Phi]e^{iS[\phi,\Phi]}\\
&=\int[D\eta]e^{i\left(S[\phi,\Phi_c]+\frac{1}{2}\left.\frac{\delta^2S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\eta^2+\mathcal{O}(\eta^3)\right)}\\
&\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\left.\frac{\delta^2S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}}\\
&=e^{iS[\phi,\Phi_c]-\frac{1}{2}\text{Tr}\ln\left(-\left.\frac{\delta^2S}{\delta\Phi}\right|_{\Phi=\Phi_c}\right)}\,,
\end{aligned}$$

where we used $\Phi=\Phi_c+\eta$ and we have defined Φ_c as the classical solution to $\left.\frac{\delta S}{\delta\Phi}\right|_{\Phi=\Phi_c}=0$. This is applicable to bosons or fermions. In general the result is a one-loop effective action of the form

$$S^{\text{eff}}_{\text{1-loop}}=ic_s\text{Tr}\ln\left(-P^2+\mathcal{M}^2+U\right)\,. \tag{2.2}$$

The constant c_s depends on the heavy field Φ. If it is a real scalar, complex scalar, Dirac fermion, gauge boson or Fadeev–Popov ghost then it takes the value $1/2, 1, -1/2, 1/2$ or -1 respectively [3]. We note that the U matrix in Eq. (2.2) is obtained after a suitable rearrangement to the required form. The relation of U to the quadratic term U' of the original Lagrangian depends on the species of Φ, i.e. on whether we are dealing with a real or complex scalar, fermion, gauge boson, and so on. For more details we refer the reader to Ref. [3]. As we will see later Refs. [3,4] have the implicit assumption that U does not contain any covariant derivatives acting openly to the right.

After evaluating the trace over spacetime by inserting a complete set of spatial and momentum eigenstates, we have a trace "tr" over internal indices (gauge, flavour, spinor, etc.):

$$S^{\text{eff}}_{\text{1-loop}}=ic_s\int d^dx\int\frac{d^dq}{(2\pi)^d}\text{tr}\ln\left(-(P_\mu-q_\mu)^2+\mathcal{M}^2+U\right)\,,$$

where $d=4-\epsilon$ in dimensional regularization. Before manipulating the logarithm to obtain an expansion in terms of higher dimension operators, we shift the momentum in the integral using the covariant derivative by inserting factors of $e^{\pm P_\mu\partial/\partial q_\mu}$:

$$\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}=ic_s\int\frac{d^dq}{(2\pi)^d}&\text{tr}\ln[e^{P_\mu\partial/\partial q_\mu}(-(P_\mu-q_\mu)^2\\
&+\mathcal{M}^2+U)e^{-P_\mu\partial/\partial q_\mu}]\,.
\end{aligned}$$

This ensures that P_μ's only appear in commutators, and the expansion will only involve manifestly gauge-covariant pieces throughout — that is the gauge field strengths, covariant derivatives and the SM fields encoded in the matrix $U(x)$:

$$\mathcal{L}^{\text{eff}}_{\text{1-loop}}=ic_s\int\frac{d^dq}{(2\pi)^d}\text{tr}\ln[-(\tilde{G}_{\nu\mu}\frac{\partial}{\partial q_\nu}+q_\mu)^2+\mathcal{M}^2+\tilde{U}]\,, \tag{2.3}$$

where

$$\begin{aligned}
\tilde{G}_{\nu\mu}&\equiv\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}\,,\\
\tilde{U}&\equiv\sum_{n=0}^{\infty}\frac{1}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}\,,
\end{aligned}$$

and we defined $G'_{\nu\mu}$ as the field strength given by $[P_\nu,P_\mu]=-G'_{\nu\mu}$. This covariant formulation is the essence of the CDE method.

In order to obtain the coefficients and structure of the higher dimension operators, there are various approaches one can take. For degenerate masses one can easily expand the action in Eq. (2.3) by integrating once its derivative with respect to the common mass scale m^2, as discussed in [3], or by making use of the Baker–Campbell–Hausdorff (BCH) formula as in [2,8]. However, for the general case of possibly non-degenerate masses, the mass matrix no longer commutes with the other matrix structures and the factorisation of the momentum integral from this structure is no longer trivial. To perform the expansion, one may use the BCH, or introduce an auxiliary parameter ξ that multiplies the diagonal mass matrix $\mathcal{M}$, defined as

$$\mathcal{M}=\xi\cdot\text{Diag}(m_i)\,, \tag{2.4}$$

[2] See Ref. [3] for a review and more technical details.