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Gauge-invariant description of Higgs phenomenon and quark confinement

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ABSTRACT

We propose a novel description for the Higgs mechanism by which a gauge boson acquires the mass. We do not assume spontaneous breakdown of gauge symmetry signaled by a non-vanishing vacuum expectation value of the scalar field. In fact, we give a manifestly gauge-invariant description of the Higgs mechanism in the operator level, which does not rely on spontaneous symmetry breaking. This enables us to discuss the confinement-Higgs complementarity from a new perspective. The "Abelian" dominance in quark confinement of the Yang–Mills theory is understood as a consequence of the gauge-invariant Higgs phenomenon for the relevant Yang–Mills–Higgs model.

1. Introduction

The *Brout–Englert–Higgs mechanism* or Higgs phenomenon for short is one of the most well-known mechanisms by which gauge bosons [1] acquire their masses [2–4]. In the conventional wisdom, the Higgs mechanism is understood in such a way that the *spontaneous symmetry breaking* (SSB) generates mass for a gauge boson: The original gauge group G is spontaneously broken down to a subgroup H by choosing a specific vacuum as the physical state from all the possible degenerate ground states (the lowest energy states). Such SSB of the original gauge symmetry is caused by a non-vanishing vacuum expectation value (VEV) $\langle \boldsymbol{\phi} \rangle \neq 0$ of a scalar field $\boldsymbol{\phi}$ governed by a given potential $V(\boldsymbol{\phi})$. For a continuous group G, there appear the massless *Nambu–Goldstone bosons* associated with the SSB $G \to H$ according to the Nambu–Goldstone theorem [5,6]. When the scalar field couples to a gauge field, however, the massless Nambu–Goldstone bosons are absorbed to provide the gauge boson with the mass. Thus, the massless Nambu–Goldstone bosons disappear from the spectrum. In a semi-classical treatment, the VEV $\langle \boldsymbol{\phi} \rangle$ is identified with one of the minima $\boldsymbol{\phi}_0$ of the scalar potential $V(\boldsymbol{\phi})$, namely, $\langle \boldsymbol{\phi} \rangle = \boldsymbol{\phi}_0 \neq 0$ with $V'(\boldsymbol{\phi}_0) = 0$.

Although this paper focuses on the Higgs phenomenon in the continuum space time, it is very instructive to learn the lattice results, because some non-perturbative and rigorous results are available on the lattice. Especially, the lattice gauge theory *à la* Wilson [7] gives a well-defined gauge theory without gauge fixing. The *Elitzur theorem* [8] tells us that the local continuous gauge symmetry cannot break spontaneously, if no gauge fixing is introduced. In the absence of gauge fixing, all gauge non-invariant Green functions vanish identically. Especially, the VEV $\langle \boldsymbol{\phi} \rangle$ of the scalar field $\boldsymbol{\phi}$ is rigorously zero,

$$\langle \boldsymbol{\phi} \rangle = 0, \tag{1}$$

no matter what the form of the scalar potential $V(\boldsymbol{\phi})$.

Therefore, we are forced to fix the gauge to cause the non-zero VEV. Even after the gauge fixing, however, we still have the problem. Whether SSB occurs or not depends on the gauge choice. For instance, in non-compact $U(1)$ gauge-Higgs model under the covariant gauge fixing with a gauge fixing parameter α, the SSB occurs $\langle \boldsymbol{\phi} \rangle \neq 0$ only in the Landau gauge $\alpha = 0$, and no SSB occur $\langle \boldsymbol{\phi} \rangle = 0$ in all other covariant gauges with $\alpha \neq 0$, as rigorously shown in [9,10]. In an axial gauge, $\langle \boldsymbol{\phi} \rangle = 0$ for compact models [11]. In contrast, it can happen that $\langle \boldsymbol{\phi} \rangle \neq 0$ in a unitary gauge regardless of the shape of the scalar potential. It is obvious that the VEV of the scalar field is not a gauge-independent criterion of SSB.

Even after breaking completely the local gauge symmetry G by imposing a suitable gauge fixing condition, there can remain a global gauge symmetry H' of G. Such a global symmetry H' is called the *remnant global gauge symmetry* [12,13]. Only a remnant global gauge symmetry H' of the local gauge symmetry G can break spontaneously to cause the Higgs phenomenon [14]. However, such a subgroup H' is not unique and the location of the breaking in the phase diagram depends on H' in the gauge-Higgs model. The relevant numerical evidences are given on a lattice [13] for different H' allowed for various confinement scenarios. Moreover, the transition occurs in the regions where the Fradkin–Shenker–Osterwalder–Seiler theorem [15,16] assures us that there

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is no transition in the phase diagram. Thus, the spontaneous gauge symmetry breaking is a rather misleading terminology.

These observations indicate that the Higgs phenomenon should be characterized in a gauge-invariant way without breaking the original gauge symmetry. In this paper, we show that a gauge boson can acquire the mass in a gauge-invariant way without assuming spontaneous breakdown of gauge symmetry which is signaled by the non-vanishing VEV of the scalar field. We demonstrate that the Higgs phenomenon occurs even without such SSB. The spontaneous symmetry breaking is sufficient but not necessary for the Higgs mechanism to work. Remember that quark confinement is realized in the unbroken gauge symmetry phase with *mass gap*. Thus, the gauge-invariant description of the Higgs mechanism can shed new light on the *complementarity* between confinement phase and Higgs phase [17].

2. Yang–Mills–Higgs model and the conventional Higgs mechanism

In this paper we use the notation for the inner product of the Lie-algebra valued quantities $\mathscr{A} = \mathscr{A}^A T_A$ and $\mathscr{B} = \mathscr{B}^A T_A$; $\mathscr{A} \cdot \mathscr{B} := 2\,\mathrm{tr}(\mathscr{A}\mathscr{B}) = \mathscr{A}^A \mathscr{B}^B 2\,\mathrm{tr}(T_A T_B) = \mathscr{A}^A \mathscr{B}^A$ under the normalization $\mathrm{tr}(T_A T_B) = \frac{1}{2}\delta_{AB}$ for the generators T_A of the Lie algebra $su(N)$ $(A = 1, 2, \dots, \dim G = N^2 - 1)$ for a gauge group $G = SU(N)$. The $SU(N)$ Yang–Mills field $\mathscr{A}_\mu(x) = \mathscr{A}_\mu^A(x) T_A$ has the field strength $\mathscr{F}_{\mu\nu}(x) = \mathscr{F}_{\mu\nu}^A(x) T_A$ defined by $\mathscr{F}_{\mu\nu} := \partial_\mu \mathscr{A}_\nu - \partial_\nu \mathscr{A}_\mu - ig[\mathscr{A}_\mu, \mathscr{A}_\nu]$.

We consider a Yang–Mills–Higgs theory specified by a gauge-invariant action. The Yang–Mills field $\mathscr{A}_\mu(x) = \mathscr{A}_\mu^A(x) T_A$ and the *adjoint scalar field* $\boldsymbol{\phi}(x) = \phi^A(x) T_A$ obey the gauge transformation:

$$\mathscr{A}_\mu(x) \to U(x)\mathscr{A}_\mu(x)U^{-1}(x) + ig^{-1}U(x)\partial_\mu U^{-1}(x),$$
$$\boldsymbol{\phi}(x) \to U(x)\boldsymbol{\phi}(x)U^{-1}(x), \quad U(x) \in G = SU(N). \tag{2}$$

For concreteness, consider the $G = SU(N)$ Yang–Mills–Higgs theory with the Lagrangian density:

$$\begin{aligned} \mathscr{L}_{\mathrm{YMH}} = &-\frac{1}{4}\mathscr{F}^{\mu\nu}(x) \cdot \mathscr{F}_{\mu\nu}(x) \\ &+ \frac{1}{2}(\mathscr{D}^\mu[\mathscr{A}]\boldsymbol{\phi}(x)) \cdot (\mathscr{D}_\mu[\mathscr{A}]\boldsymbol{\phi}(x)) \\ &- V(\boldsymbol{\phi}(x) \cdot \boldsymbol{\phi}(x)), \end{aligned} \tag{3}$$

where we have defined the covariant derivative $\mathscr{D}_\mu[\mathscr{A}] := \partial_\mu - ig[\mathscr{A}_\mu, \cdot]$ in the adjoint representation. We assume that the adjoint scalar field $\boldsymbol{\phi}(x) = \phi^A(x) T_A$ has the fixed radial length, which is represented by a constraint[1]:

$$\boldsymbol{\phi}(x) \cdot \boldsymbol{\phi}(x) \equiv \boldsymbol{\phi}^A(x)\boldsymbol{\phi}^A(x) = v^2. \tag{4}$$

Notice that $\boldsymbol{\phi}(x) \cdot \boldsymbol{\phi}(x)$ is a gauge-invariant combination. Therefore, the potential V as an arbitrary function of $\boldsymbol{\phi}(x) \cdot \boldsymbol{\phi}(x)$ is invariant under the gauge transformation. The covariant derivative $\mathscr{D}_\mu[\mathscr{A}] := \partial_\mu - ig[\mathscr{A}_\mu, \cdot]$ transforms according to the adjoint representation under the gauge transformation: $\mathscr{D}_\mu[\mathscr{A}] \to U(x)\mathscr{D}_\mu[\mathscr{A}]U^{-1}(x)$. This is also the case for the field strength $\mathscr{F}_{\mu\nu}(x)$. Moreover, the constraint (4) is invariant under the gauge transformation and does not break the gauge invariance of the theory. Therefore, $\mathscr{L}_{\mathrm{YMH}}$ of (3) with the constraint (4) is invariant under the local gauge transformation (2).

For $N = 2$, this theory is nothing but the well-known Georgi–Glashow model which exemplifies the SSB of the local gauge symmetry from $SU(2)$ down to $U(1)$ except for the magnitude of the scalar field being fixed (4). In this paper, we focus our discussions on the $SU(2)$ case.

First, we recall the conventional description for the Higgs mechanism. If the scalar field $\boldsymbol{\phi}(x)$ acquires a non-vanishing VEV $\langle\boldsymbol{\phi}(x)\rangle = \langle\boldsymbol{\phi}\rangle$, then the covariant derivative reduces to

$$\begin{aligned} \mathscr{D}_\mu[\mathscr{A}]\phi(x) &:= \partial_\mu\boldsymbol{\phi}(x) - ig[\mathscr{A}_\mu(x), \boldsymbol{\phi}(x)] \\ &\to -ig[\mathscr{A}_\mu(x), \langle\boldsymbol{\phi}\rangle] + \dots, \end{aligned} \tag{5}$$

and the Lagrangian density reads

$$\begin{aligned} \mathscr{L}_{\mathrm{YMH}} \to &-\frac{1}{2}\,\mathrm{tr}_G\{\mathscr{F}^{\mu\nu}(x)\mathscr{F}_{\mu\nu}(x)\} \\ &- g^2\,\mathrm{tr}_G\{[\mathscr{A}^\mu(x), \langle\boldsymbol{\phi}\rangle][\mathscr{A}_\mu(x), \langle\boldsymbol{\phi}\rangle]\} + \dots \\ = &-\frac{1}{2}\,\mathrm{tr}_G\{\mathscr{F}^{\mu\nu}(x)\mathscr{F}_{\mu\nu}(x)\} \\ &- g^2\,\mathrm{tr}_G\{[T_A, \langle\boldsymbol{\phi}\rangle][T_B, \langle\boldsymbol{\phi}\rangle]\}\mathscr{A}^{\mu A}(x)\mathscr{A}_\mu^B(x) + \dots \end{aligned} \tag{6}$$

To break spontaneously the local continuous gauge symmetry G by realizing the non-vanishing VEV $\langle\boldsymbol{\phi}\rangle$ of the scalar field $\boldsymbol{\phi}$, we choose the *unitary gauge* in which the scalar field $\boldsymbol{\phi}(x)$ is pointed to a specific direction $\boldsymbol{\phi}(x) \to \boldsymbol{\phi}_\infty$ uniformly over the spacetime.

This procedure does not completely break the original gauge symmetry G. Indeed, there may exist a subgroup H of G such that $\boldsymbol{\phi}_\infty$ does not change under the local H gauge transformation. This is the *partial SSB* $G \to H$: the mass is provided for the coset G/H (broken parts), while the mass is not supplied for the H-commutative part of $\mathscr{A}_\mu$:

$$\begin{aligned} \mathscr{L}_{\mathrm{YMH}} \to &-\frac{1}{2}\,\mathrm{tr}_G\{\mathscr{F}^{\mu\nu}(x)\mathscr{F}_{\mu\nu}(x)\} \\ &- (gv)^2 \mathrm{tr}_{G/H}\{\mathscr{A}^\mu(x)\mathscr{A}_\mu(x)\}. \end{aligned} \tag{7}$$

After the partial SSB, therefore, the resulting theory is a gauge theory with the residual gauge group H.

For $G = SU(2)$, by taking the usual *unitary gauge* in which the scalar field $\boldsymbol{\phi}(x) = \phi^A(x) T_A$ $(A = 1, 2, 3)$ is chosen so that

$$\langle\boldsymbol{\phi}_\infty\rangle = vT_3, \quad \text{or} \quad \langle\phi_\infty^A\rangle = v\delta^{A3}, \tag{8}$$

the second term of (6) generates the mass term,

$$\begin{aligned} &-g^2v^2\,\mathrm{tr}_G\{[T_A, T_3][T_B, T_3]\}\mathscr{A}^{\mu A}(x)\mathscr{A}_\mu^B(x) \\ &= \frac{1}{2}g^2v^2(\mathscr{A}^{\mu 1}(x)\mathscr{A}_\mu^1(x) + \mathscr{A}^{\mu 2}(x)\mathscr{A}_\mu^2(x)). \end{aligned} \tag{9}$$

For $SU(2)$, indeed, the off-diagonal gluons $\mathscr{A}_\mu^a$ $(a = 1, 2)$ acquire the same mass $M_W := gv$, while the diagonal gluon $\mathscr{A}_\mu^3$ remains massless. Even after taking the unitary gauge (8), $U(1)$ gauge symmetry described by $\mathscr{A}_\mu^3$ still remains as the residual local gauge symmetry $H = U(1)$, which leaves $\boldsymbol{\phi}_\infty$ invariant (the local rotation around the axis of the scalar field $\boldsymbol{\phi}_\infty$).

Thus, the SSB is sufficient for the Higgs mechanism to take place. But, it is not clear whether the SSB is necessary or not for the Higgs mechanism to work.

In the complete SSB $G \to H = \{1\}$, all components of the Yang–Mills field become massive with no massless components:

[1] After imposing the constraint (4), the subsequent argument should hold irrespective of the form of the potential V. The vacuum manifold in the target space of the scalar field is determined by the minima of the potential V, which also satisfies the constraint (4). However, there are some options as to when and how the constraint is incorporated, see e.g., [39]. The potential is omitted in what follows when any confusion does not occur. Moreover, this model is perturbatively non-renormalizable and the non-perturbative treatment is required.