Accepted Manuscript

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PII: S0370-2693(16)30532-9
DOI: http://dx.doi.org/10.1016/j.physletb.2016.09.037
Reference: PLB 32293

To appear in: *Physics Letters B*

Received date: 19 September 2016
Accepted date: 19 September 2016

Please cite this article in press as: A. Majhi, Conformal blocks on a 2-sphere with indistinguishable punctures and implications on black hole entropy, *Phys. Lett. B* (2016), http://dx.doi.org/10.1016/j.physletb.2016.09.037

Conformal blocks on a 2-sphere with indistinguishable punctures and implications on black hole entropy

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The dimensionality of the Hilbert space of a Chern-Simons theory on a 3-fold, in the presence of Wilson lines carrying spin representations, had been counted by using its link with the Wess-Zumino theory, with level k, on the 2-sphere with points (to be called punctures) marked by the piercing of the corresponding Wilson lines and carrying the respective spin representations. It is shown, in the weak coupling (large k) limit, the formula decouples into two characteristically distinct parts; one mimics the dimensionality of the Hilbert space of a collection of non-interacting spin systems and the other is an effective overall correction contributed by all the punctures. The exact formula yield from this counting has been shown earlier to have resulted from the consideration of the punctures to be distinguishable. We investigate the same counting problem by considering the punctures to be indistinguishable. Although the full formula remains undiscovered, nonetheless, we are able to impose the relevant statistics for indistinguishable punctures in the approximate formula resulting from the weak coupling limit. As an implication of this counting, in the context of its relation to that of black hole entropy calculation in quantum geometric approach, we are able to show that the logarithmic area correction, with a coefficient of $-3/2$, that results in this method of entropy calculation, in independent of whether the punctures are distinguishable or not.

I. INTRODUCTION

The dimension of the Hilbert space of the Chern-Simons(CS) theory on a 3-fold with boundary 2-sphere is given by the number of conformal blocks of an $SU(2)_k$ Wess-Zumino(WZ) theory of level k on that boundary[1]. In particular, in the presence of Wilson lines carrying spin representations in the 3-fold, which pierce the boundary 2-sphere producing marked points (henceforth, to be called punctures) carrying the corresponding spin representations, the dimension of the CS theory is then given by the number of conformal blocks of the WZ theory on the 2-sphere with punctures. This counting was explicitly done in [2] to yield this number $\Omega(j_1, j_2, \cdots, j_p)$ (say) for a set of p punctures carrying spins $j_1, j_2, \cdots, j_p$.

Now, if one wishes to count this number for all possible spins, then one does a sum over all possible spins i.e. $\sum_{j_1,\cdots,j_p} \Omega(j_1, j_2, \cdots, j_p)$, which can be recast as a sum over all possible spin configurations i.e. $\sum_{\{s_j\}} \Omega[\{s_j\}]$ by the use of multinomial expansion (a spin configuration $\{s_j\}$ constitutes a set such that there are s_j number of spin j)[3]. This yields a formula for $\Omega[\{s_j\}]$ which manifests the number of conformal blocks of the WZ theory on a punctured sphere with spin configuration $\{s_j\}$, if the punctures are considered to be distinguishable.

Here, we investigate this counting problem by considering the punctures to be indistinguishable. Unfortunately, in this case, we do not have a way, unlike the multinomial expansion, to go over from a set of spins to spin configurations simply because we do not know how to take the sum over spins[1]. Also, the way in which the counting exercise originates in terms of the fusion matrices of the WZ theory [2] it is not clear at the moment how to differentiate between the counting methods for distinguishable and indistinguishable punctures. So, we do an approximation to the formula for $\Omega(j_1, \cdots, j_p)$ and express it in a convenient form where the imposition of the statistics, depending on the disnguishability of the punctures, becomes manifestly trivial. Consequently, we arrive at an effective formula for the $\Omega[\{s_j\}]$ for indistinguishable punctures. An immediate application of this approximate result is found in the context of black hole entropy calculation in the quantum geometric approach. We show that the subleading logarithmic area correction of the entropy is independent of whether the punctures are distinguishable or not.

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[1] This particular problem even exists in the elementary statistical mechanics of a collection of particles[7].