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Conductivity bound from dirty black holes

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We propose a lower bound of the dc electrical conductivity in strongly disordered, strongly interacting quantum field theories using holography. We study linear response of black holes with broken translational symmetry in Einstein–Maxwell-dilaton theories of gravity. Using the generalized Stokes equations at the horizon, we derive the lower bound of the electrical conductivity for the dual two dimensional disordered field theory.

1. Introduction

Based on experimental results, studying some quantum systems in condensed matter theory such as strange metals and cold atomic phase one needs to consider strongly interacting many-body quantum physics [1]. Quantum critical physics has important role in describing macroscopic observables in these systems [2]. One of the important observables is the electrical conductivity at finite density and disorder. To study such strongly interacting systems, the usual theoretical methods are not efficient and one should use new methods which are based on the nonperturbative approaches.

Using the gauge-string correspondence is a new tool for studying the transport coefficients in strongly interacting systems. In this way, the quantum dynamics is encoded in the classical gravity in an asymptotically AdS spacetime. To consider finite temperature systems, one should add a black hole in the bulk space. Such systems in the condensed matter physics have large N matrix degrees of freedom. In this paper we are interested in studying the electrical DC conductivity at finite density and disorder. It is well known that a charged fluid with Galilean-invariant symmetry has infinite electrical conductivity. It is consequence of momentum conservation in the theory. Hence, all currents will have a non-zero conserved momentum and so they will not decay. However, recent studies from numerical holography in systems with breaking translation symmetry exhibit finite conductivity [3–6]. Recent studies of the DC conductivity have been done for a charged fluid on the black hole horizon [7–10]. One should notice that it is related to the near horizon geometry of black holes.

To study strongly interacting quantum disorder systems from holography, massive gravity has been studied in [11,12]. In such systems momentum relaxation is achieved even though translations are not explicitly broken in the bulk of the geometry. One important prediction of these models is that the disorder alone does not derive a metal–insulator transition. In mean-field disorder systems, insulators are not disorder-driven. The formation of a gap which should be proportional to the amount of disorder or additional localized features was studied in [13]. In such studies the effects of disorder on a holographic superconductor have been investigated by introducing a random chemical potential on the boundary [14,15]. It was shown that increasing disorder leads to increasing the superconducting order and subsequently to the transition to a metal.

Recently, absence of disorder-driven metal–insulator transitions has been studied from holography [16]. It was found that the electric DC conductivity of simple holographic disorder systems is bounded by the following universal value

$$\sigma \geq 1. \tag{1}$$

This quantity σ is measured in units of $\frac{e^2}{\hbar}$, with e the $U(1)$ charge of the carriers.[1] This bound means that one can not get an insulating phase by disorder-driven. They consider Einstein–Maxwell theory on AdS_4 without any free parameter and show that the

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[1] We consider $\frac{e^2}{\hbar} = 1$.

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bound does not depend on the temperature or fluctuations in the charge density. The simple holographic model in [16] means that there is no coupling between scalar dilaton ϕ to the Maxwell field also no additional charged fields in the bulk of the theory.

In this paper we extend the results of [16] and obtain the bound for DC conductivity of disordered Einstein–Maxwell-dilaton (EMD) holographic systems. We bound σ in a relativistic theory which is dual to a disorder black hole in EMD geometry and in two spatial dimension. In such holographic models the field theory is deformed by a charge-neutral relevant scalar operator. In general EMD holographic models, the DC conductivity σ can be either metallic or insulating [18–20]. Such models are interesting to study the strange metal phase of high temperature superconductors.

We have studied before two important properties of the strange metals, the Ohmic resistivity and the inverse Hall angle, in the presence of finite-coupling corrections in [21]. In this study, we considered the AdS spacetime in the light-cone frame. This frame could be used to study physical properties of strange metals [22]. The electrical DC conductivity of massive $\mathcal{N}=2$ hypermultiplet fields at finite temperature and in an $\mathcal{N}=4$ $SU(N_c)$ super-Yang–Mills theory in the large N_c and finite-coupling correction was studied in [23].

This paper is organized as follows. In the next section we review the EMD gravity in [10] and study the charged horizon fluid. We also define heat and electric currents in the horizon. In section two we use variational method and derive the bound on the conductivity. In the last section we discuss and summarize our results.

2. Dirty black holes

The EMD holographic theories in D spacetime dimensions have been studied recently in [10]. In the following the case of AdS_4 has been considered, also some notations of [10] have been changed. In the following we discuss the general class of holographic lattices. Periodic lattices with chemical potential and real scalar field have been studied in [3,24]. Also Q-lattice models with more scalar fields were studied in [25–27]. The helical lattices were studied in [8,9,17]. Study of holographic lattice models in the presence of magnetic fields has been done in [28–30]. The generalization of the results of this section in the presence of magnetic fields has been done in [30].

The action is given by

$$S=\int d^4x\sqrt{-g}\left(R-V(\phi)-\frac{Z(\phi)}{4}F^2-\frac{1}{2}(\partial\phi)^2\right). \tag{2}$$

Here, F is the Maxwell field strength of A which is dual of a global $U(1)$ gauge field. There is also an operator dual to the scalar dilaton field ϕ. One should assume $V(\phi=0)=-6, V'(\phi=0)=0$ and $Z(\phi=0)=1$ to find a unit radius AdS_4 geometry. We have set the AdS_4 radius to unity, as well as setting $16\pi G=1$. The equations of motion are given by

$$\begin{aligned}
&R_{\mu\nu}-\frac{V(\phi)}{2}g_{\mu\nu}-\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\\
&\quad-\frac{1}{2}Z(\phi)\left(F_{\mu\rho}F_\nu{}^\rho-\frac{1}{4}g_{\mu\nu}F^2\right)=0,\\
&\nabla_\mu\left[Z(\phi)F^{\mu\nu}\right]=0,\\
&\nabla^2\phi-V'(\phi)-\frac{1}{4}Z'(\phi)F^2=0.
\end{aligned} \tag{3}$$

We consider a general static electrically charged black hole geometry as

$$ds^2=-U(r)V(r,\mathbf{x})\,dt^2+\frac{W(r,\mathbf{x})}{U(r)}dr^2+G_{ij}dx_idx_j, \tag{4}$$

with a gauge-field A given by

$$A=a_t(r,\mathbf{x})\,dt=\Phi(r,\mathbf{x})\,dt. \tag{5}$$

Here G_{ij} is a metric on a two dimensional manifold at fixed r. The holographic direction is denoted by r and the boundary field theory directions are $(t,\vec{x})$. A connected black horizon is located at $r=0$. The dual field theory temperature T is given by the Hawking temperature of black hole. In the UV boundary condition, as $r\to\infty$, the solutions are taken to approach AdS_4 spacetime. The gauge field also goes to the spatially dependent chemical potential $\mu(x)$ in the boundary. To consider periodic lattices, one assumes periodic conditions for fields in the boundary. The period in spatial directions is denoted by L_i and the geometry at fixed r parameterizes a torus with period $x_i\sim x_i+L_i$ also the black hole horizon has the same topology. With using Kruskal coordinate $v=t+\frac{\ln r}{4\pi T}+\ldots$, one finds the near horizon expansions of the metric functions and fields as

$$\begin{aligned}
U(r)&=4\pi T\,r+U^{(1)}r^2+\ldots,\\
a_t(r,x)&=a_t^{(0)}W^{(0)}(x)\,r+a_t^{(1)}(x)\,r^2+\ldots,\\
W(r,x)&=W^{(0)}(x)+W^{(1)}(x)\,r+\ldots,\\
V(r,x)&=W^{(0)}(x)+V^{(1)}(x)\,r+\ldots,\\
G_{ij}&=G_{ij}^{(0)}+G_{ij}^{(1)}r+\ldots,\\
\phi(x)&=\phi^{(0)}(x)+r\phi^{(1)}(x)+\ldots.
\end{aligned} \tag{6}$$

One should notice that $W^{(0)}(x)=V^{(0)}(x)$.

The electric charge density at the horizon is $\rho_h=\sqrt{-g}Z(\phi)F^{tr}|_h=\sqrt{-g_0}a_t^{(0)}Z^{(0)}$ where $Z^{(0)}\equiv Z(\phi^{(0)}(x))$ and $a^{(0)}\equiv a(\phi^{(0)}(x))$. The scalar dilaton value at the horizon is denoted as $\phi^{(0)}$. Henceforth, we use such notation for values of parameters near the horizon.

By turning on electric current E and temperature gradient ζ on the geometry at fixed r, the black hole will response [26]. One should assume $E_i=E_i(x)$, $\zeta_i=\zeta_i(x)$ and use the appropriate linear perturbations $\delta g_{\mu\nu},\delta a_\mu$, $\delta\phi$ which are functions of (r,x^i) [10]. At the black hole horizon, the leading order terms are

$$\begin{aligned}
&\delta g_{tt}\to U(r)\,\delta g_{tt}^{(0)}(x),\quad \delta g_{rr}\to\frac{\delta g_{rr}^{(0)}(x)}{U},\\
&\delta g_{ij}\to\delta g_{ij}^{(0)}(x),\quad \delta g_{tr}\to\delta g_{tr}^{(0)}(x),\\
&\delta g_{ti}\to\delta g_{ti}^{(0)}(x)-V(r,\mathbf{x})U(r)\zeta_i\frac{\ln r}{4\pi T},\quad \delta g_{ri}\to\frac{1}{U}\left(\delta g_{ri}^{(0)}(x)\right),\\
&\delta a_t\to\delta a_t^{(0)}(x),\quad \delta a_r\to\frac{1}{U}\left(\delta a_r^{(0)}(x)\right),\\
&\delta a_i\to\frac{\ln r}{4\pi T}(-E_i+a_t(r,\mathbf{x})\zeta_i),
\end{aligned} \tag{7}$$

with the following constraints on the leading order terms of x as

$$\delta g_{tt}^{(0)}+\delta g_{rr}^{(0)}-2\,\delta g_{rt}^{(0)}=0,\qquad \delta g_{ri}^{(0)}=\delta g_{ti}^{(0)},\qquad \delta a_r^{(0)}=\delta a_t^{(0)}. \tag{8}$$

2.1. The electric and heat current

The bulk electric current density is defined as

$$J^i=\sqrt{-g}Z(\phi)F^{ir}. \tag{9}$$

At linearized order for the perturbed black holes, one finds