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Model building with non-compact cosets

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ABSTRACT

We explore Goldstone boson potentials in non-compact cosets of the form $SO(n,1)/SO(n)$. We employ a geometric approach to find the scalar potential, and focus on the conditions under which it is compact in the large field limit. We show that such a potential is found for a specific misalignment of the vacuum. This result has applications in different contexts, such as in Composite Higgs scenarios and theories for the Early Universe. We work out an example of inflation based on a non-compact coset which makes predictions which are consistent with the current observational data.

1. Introduction

Goldstone bosons are popular actors in theories beyond the Standard Model of Particle Physics. They resolve the dichotomy between the aptness of scalars in cosmological theories and the theoretical hierarchy problems that fundamental scalars suffer. The study of their Effective Field Theory is further motivated by their omnipresence in UV theories with global symmetries, such as models for axions [1–3], and supersymmetry [4–6].

There is a vast body of literature which focuses on Goldstone bosons in compact cosets, that is, on theories in which a compact global symmetry breaks spontaneously to its compact subgroup. An example is the Minimal Composite Higgs Model MCHM_5, in which $SO(5) \to SO(4)$. In theories of this kind the Goldstone bosons lie on a compact manifold, such as the hypersphere $S^4 \simeq SO(5)/SO(4)$. Their interactions are invariant under a shift symmetry, such that a potential is forbidden at all orders in perturbation theory.

In the presence of a source of explicit breaking of the global group the shift symmetry is broken, and a potential for the pseudo-Goldstone Bosons (pGBs) may be generated. Such an explicit breaking can be mediated by external gauge bosons which gauge part of the global group, as is common in Composite Higgs models, or by couplings to instantons which do not respect the symmetry, as is the case with axions. The resulting potential will have a remnant periodic shift symmetry, stabilizing it against quantum corrections. Examples which employ such a scenario are Composite Higgs models [7], Natural Inflation [8], Goldstone Inflation [9,10], and composite dark matter [11].

Goldstone bosons in non-compact cosets have received far less attention. Of particular interest are models in which a non-compact group breaks to its compact subgroup. There are indications that such cosets could give promising models of inflation [12] and electroweak symmetry breaking [13]. Like in the compact case, these cosets may address hierarchy problems by giving rise to stable scalar potentials.

Here we will explore the idea that scalar sectors can be studied in a coordinate-invariant way, something that has recently attracted some attention in the context of Higgs Effective Field Theory [14,13,15]. It has been observed [16] that results in non-compact cosets may be extrapolated from corresponding compact cosets by considering imaginary parameters, such that the corresponding manifold undergoes a Wick rotation. Here we instead follow a more general, geometric approach to study the potential of the Goldstone bosons of the hyperbolic space $SO(n,1)/SO(n)$. In section 2 we describe the different models for hyperboloids that are of interest to this analysis.

The shift symmetry in the non-compact case will also be broken in the presence of explicit symmetry breaking effects, misaligned with the original breaking. For $SO(n,1)/SO(n)$ the remnant symmetry takes the form of a discrete scaling symmetry. We will parametrize the explicit breaking without choosing a particular particle physics interpretation, bearing in mind the different ways of breaking the shift symmetry. Our approach generalizes the analysis of [12] in the context of inflation, and provides an alternative description of the discussion of Goldstone bosons in non-compact cosets in [15].

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The focus of this paper will be on the conditions under which the Goldstone boson potential is bounded, i.e. confined to lie in a specific region in the limit in which the field excursion of the scalars is large. This is of particular interest for inflationary model building, as in typical scenarios one has to explain the gap between the magnitude of the scalar potential ($V^{1/4} \sim 10^{15}$ GeV) and the large field excursion ($\Delta\phi \sim M_p$), highlighted by the familiar Lyth bound [17]. In section 3 we will show that a bounded potential is generated when the symmetry breaking parameters transform as a null vector of the hyperbolic space.

In the last section we will discuss the application of this class of models to inflation. We will explore the inflationary predictions, and compare them to data from the Planck collaboration [18].

2. Models of hyperbolic space

Below the scale of the spontaneous breaking $SO(n,1) \to SO(n)$, the relevant degrees of freedom are a set of Goldstone bosons which lie on the non-compact, n-dimensional hyperbolic sheet given by $SO(n,1)/SO(n)$. In the absence of any additional sources of breaking, the Goldstone bosons respect a shift symmetry which forbids a scalar potential. They will obtain a potential when they couple to a source of explicit breaking. This is for instance the case if a smaller group is gauged by external bosons such as in Composite Higgs models. This case is well studied; it has for instance recently been discussed in [13] in the context of Higgs Effective Field Theory. Here we use a less restrictive approach, in which we focus on the transformation properties of the symmetry breaking parameters which couple to the Goldstone bosons.

The coset $SO(n,1)/SO(n)$ can be described as a sheet of a space-like hyperbola,[1] defined by the interval

$$L = \{(x_1, \dots, x_{n+1})) : x_{n+1}^2 - x_n^2 - x_{n-1}^2 - \dots - x_1^2 = \ell^2 \text{ and } x_{n+1} > 0\} \tag{1}$$

$$ds_L^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta = dx_{n+1}^2 - \sum_{i=1}^{n} dx_i^2 \tag{2}$$

where $x_{n+1} > 0$. This space is associated with the Hermitian form or dot product with the signature $(n,1)$,

$$g_{\mu\nu} x^\mu y^\nu = x_\mu y^\mu = -x_1 y_1 + x_2 y_2 + \dots + x_n y_n \tag{3}$$

It has constant negative curvature,

$$\mathcal{R}_{fieldspace} = n(1-n) < 0. \tag{4}$$

As we will see in the following, the model "L" is not always the most transparent choice to describe the features of the Goldstone potential. An alternative choice is the Poincare disk model, which is defined by

$$J = \{(x_1, \dots, x_{n+1})) : x_1^2 + \dots + x_{n+1}^2 = \ell^2 \text{ and } x_{n+1} > 0\} \tag{5}$$

$$ds_J^2 = \frac{dx_{n+1}^2 + \sum_{i=1}^{n} dx_i^2}{x_{n+1}^2} \tag{6}$$

This model is related to "L" by a central projection from the point $(-\ell, 0, \dots, 0)$ (Fig. 1),

$$L \to J,$$
$$(x_1, \dots, x_n, x_{n+1}) \mapsto (x_1\ell/x_{n+1}, \dots, x_n\ell/x_{n+1}, \ \ell^2/x_{n+1}) \tag{7}$$

[1] The terminology in this chapter is adopted often in analogy with space-time symmetries, however, the reader is assured that we consider internal symmetries only in this paper.

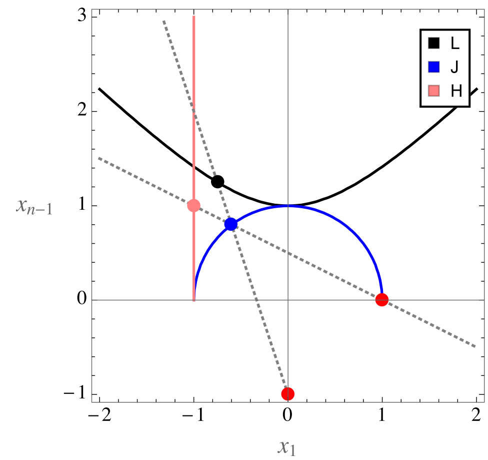

Fig. 1. *Models of the coset:* in two dimensions. An arbitrary point on the hyperbola, $\{x_1^L, x_{n+1}^L\}$ (in black), can be projected it unto the sphere $x_1^2 + x_{n+1}^2 = 1$ from the point $\{-1, 0\}$. This gives the coordinates in the Disk model, $\{x_1^J, x_{n+1}^J\} = \left\{\frac{x_1^L}{x_{n+1}^L}, \frac{1}{x_{n+1}^L}\right\}$ (in blue). A further projection from the point $\{1, 0\}$ onto the line $x_1 = -1$ gives the coordinates in "H" $\{x_1^H, x_{n+1}^H\} = \left\{-1, 2/\left(x_{n+1}^L\left(1 - \frac{x_1^L}{x_{n+1}^L}\right)\right)\right\}$ (in pink). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Another choice is the Poincare Half plane model, which reduces to the well known complex projective coordinates often employed in supersymmetry for $n = 2$. The Half plane model is defined by

$$H = \{(1, x_2, \dots, x_{n+1})) : x_{n+1} > 0\} \tag{8}$$

$$ds_H^2 = \frac{dx_{n+1}^2 + \sum_{i=2}^{n} dx_i^2}{x_{n+1}^2} \tag{9}$$

The Half plane model can in turn be related to J by a central projection from the point $(0, \dots, 0, \ell)$ (Fig. 2), i.e. the mapping

$$J \to H,$$
$$(x_1, \dots, x_n, x_{n+1})$$
$$\mapsto (-\ell, \ 2\ell x_2/(x_1 - \ell), \dots, \ 2\ell x_{n+1}/(x_1 - \ell)) \tag{10}$$

From this, it follows that "H" and "L" are related by the mapping,

$$L \to H,$$
$$(x_1, \dots, x_n, x_{n+1})$$
$$\mapsto (-\ell, \ 2\ell x_2/(x_1 - x_{n+1}), \dots, \ 2\ell^2/(x_1 - x_{n+1})) \tag{11}$$

3. Compact potentials from non-compact cosets

Before the second symmetry breaking, the Goldstone bosons are massless and their target metric is described by the hyperboloid $SO(n,1)/SO(n)$. In the previous section we have shown different ways to describe such a field space.

A potential for the Goldstone bosons of the coset $SO(n,1)/SO(n)$ is generated in the presence of symmetry breaking effects, misaligned from the original vacuum. We use a very minimal description, based on particular choices for the transformation properties of the symmetry breaking parameters under the higher dimensional Lorentz group. Here we derive which transformation properties lead to a compact potential, i.e., a potential that does not diverge in the large field limit.