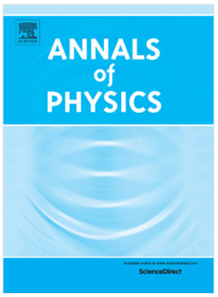


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# Extended Hamiltonians and shift, ladder functions and operators

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**Abstract**

In recent years, many natural Hamiltonian systems, classical and quantum, with constants of motion of high degree, or symmetry operators of high order, have been found and studied. Most of these Hamiltonians, in the classical case, can be included in the family of extended Hamiltonians, geometrically characterized by the structure of warped manifold of their configuration manifold. For the extended Hamiltonians, the characteristic constants of motion of high degree are polynomial in the momenta of determined form. We consider here a different form of the constants of motion, based on the factorization procedure developed by S. Kuru, J. Negro and others. We show that an important subclass of the extended Hamiltonians admits factorized constants of motion and we determine their expression. The classical constants may be non-polynomial in the momenta, but the factorization procedure allows, in a type of extended Hamiltonians, their quantization via shift and ladder operators, for systems of any finite dimension.

*Keywords:* superintegrable systems, extended Hamiltonians, Schrödinger operators, Laplace-Beltrami operators, shift and ladder, quantization, factorization, warped manifolds
*2010 MSC:* 81S05, 81R12, 81R15, 70H06

## 1. Introduction

In the past few years, the systematic study of classical Hamiltonian systems admitting polynomial constants of motion of high degree, and quantum Hamiltonian operators with symmetry operators of high order, produced several articles [6, 4, 16, 17, 20, 21, 22, 23, 24, 26], many of them focused on the superintegrability of the systems considered. In several articles [7, 8, 9, 10, 11, 12, 13, 14], we, together with Luca Degiovanni, were able to unify most of those Hamiltonian systems under the common structure of *extended Hamiltonians*. For

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