Annals of Physics 380 (2017) 12–22

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Compressible fluids with Maxwell-type equations, the minimal coupling with electromagnetic field and the Stefan–Boltzmann law

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HIGHLIGHTS

- Higher-derivative Lagrangian for a charged fluid.
- Electromagnetic coupling and Dirac's constraint analysis.
- Partition function through path integral formalism.
- Stefan–Boltzmann-kind law through the partition function.

ARTICLE INFO

Article history:
Received 8 July 2016
Accepted 26 February 2017
Available online 16 March 2017

Keywords:
Compressible fluid
Electromagnetic background
Stefan–Boltzmann law

ABSTRACT

In this work we have obtained a higher-derivative Lagrangian for a charged fluid coupled with the electromagnetic fluid and the Dirac's constraints analysis was discussed. A set of first-class constraints fixed by noncovariant gauge condition were obtained. The path integral formalism was used to obtain the partition function for the corresponding higher-derivative Hamiltonian and the Faddeev–Popov ansatz was used to construct an effective Lagrangian. Through the partition function, a Stefan–Boltzmann type law was obtained.

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http://dx.doi.org/10.1016/j.aop.2017.02.017
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1. Introduction

In recent papers the authors have discussed that, as an alternative way for the description of fluid dynamics, concerning both the compressible fluids [1] and the equations of plasma [2], the better path would be through the recasting of the equations of motion to obtain a set of Maxwell-type equations for the fluid. This transformation in the structure of the equations of motion results in the generalization of the concept of charge and current connected to the dynamics of the fluid [3,4]. The identification of what will be considered as a source term in the resulting theory depends on the choice of the objects which will form the main part of its new structure of fluid dynamics. In Lighthill's work concerning the sound radiated by a fluid flow [5], the applied stress tensor was considered as the source of the radiation field. R.J. Thompson [2] recently introduced an extension of this new structure of the plasma equations of motion, for each kind of fluid, from the equations of motion that describe such system. The reason is to understand thermodynamical arguments in order to obtain how the energy density ρ depends on the temperature T for a fluid's equation of state given by $p = \omega\rho$. Besides, the Stefan–Boltzmann law has been widely discussed in the scenario of black holes thermodynamics [6], from where we know that the energy density is inversely proportional to the temperature. More recently, the observed acceleration of the Universe demands the existence of a new component, the termed dark energy, which rules out all other forms of energy and has a negative pressure. The presence of such energy in the Universe deserves detailed analysis, such as the consequences related to the application of the generalized second law [7] or the entropy bound [8]. Some elements, such as the phantom field ($\omega < -1$), fields with a negative kinetic energy, negative temperature and the entropy being always positive, can change completely the evolution of black holes and their connection to the generalized second law, as was discussed in [9,10].

With these motivations in mind, our proposal here, besides to consider a charged fluid, which is defined by the Lagrangian density (1) below, is to discuss its coupling with the electromagnetic field and its corresponding constraints through Dirac's constraint classification. Another important point will be the construction of the theory partition function after determining its constraint structure [11], which allows us, finally, to evaluate all the thermodynamical quantities.

The work is organized in such a way that in Section 2 we have a description of the general aspects of the theory's canonical structure. In Section 3 we present the transition amplitude using the path integral formulation. In Section 4 we will analyze the theory in thermodynamical equilibrium using the imaginary-time formalism and we will derive the Stefan–Boltzmann law and finally in the last section we present the conclusions.

2. Canonical structure

Recently [12], some of us have shown that a Lagrangian formulation for a compressible fluid can be obtained, analogously to the one described by Marmanis concerning an incompressible fluid [3], resulting in a Maxwell-type action for the fluid considering the viscosity, given by

$$\mathcal{L}_{fluid} = -\frac{1}{4} T_{\mu\nu} T^{\mu\nu}, \tag{1}$$

where $T_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$ is the strength tensor of the fluid. The four-vector potential $U_\mu \equiv (U_0, \vec{U})$, where U_0 is the energy function and $\vec{U}$ is the average velocity field [12]. The space–time metric is $\eta_{\mu\nu} = (- + ++)$.

From the Lagrangian density in Eq. (1) we can compute the momentum $\vec{\pi}$ which is the negative of the Lamb vector, given by $\vec{\omega} \times \vec{U}$, ω is the vorticity. The result is the well known Navier–Stokes equation given by

$$\frac{\partial \vec{U}}{\partial t} + \vec{\omega} \times \vec{U} + \nabla\left(\frac{1}{2}U^2\right) = -\frac{1}{\rho}\nabla p,$$