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Non-local currents and the structure of eigenstates in planar discrete systems with local symmetries

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HIGHLIGHTS

- We extend the framework of non-local currents to discrete planar systems.
- Structural information about the eigenstates is gained.
- Conditions for the constancy of non-local currents are derived.
- We use the framework to design two types of example systems featuring locally symmetric eigenstates.

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ABSTRACT

Local symmetries are spatial symmetries present in a subdomain of a complex system. By using and extending a framework of so-called non-local currents that has been established recently, we show that one can gain knowledge about the structure of eigenstates in locally symmetric setups through a Kirchhoff-type law for the non-local currents. The framework is applicable to all discrete planar Schrödinger setups, including those with non-uniform connectivity. Conditions for spatially constant non-local currents are derived and we explore two types of locally symmetric subsystems in detail, closed-loops and one-dimensional open

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ended chains. We find these systems to support locally similar or even locally symmetric eigenstates.

1. Introduction

Local symmetries, i.e. symmetries that are only present in a spatial part of a given system, are ubiquitous in nature, a popular example being quasicrystals [1–5]. Due to the long-range order of quasicrystals, one can always find structures of equal structure which can be described by local symmetries. Other examples are large molecules [6,7] and, in general, systems where the global symmetry is broken due to defects. Beyond this, a second class of systems are those which are specifically designed in such a way that they possess local symmetries. Examples therefore are photonic multilayers [8–11] or photonic waveguide arrays [12–15].

Despite their widespread presence in both natural and artificial physical systems, a systematic and in-depth treatment of the influence of local symmetries on a system's behaviour is still missing. A reason for this might lie in the tools used. In quantum systems, for example, the treatment of symmetries is based on the determination of the Hamiltonian group, i.e. the set of operators commuting with the Hamiltonian of the considered system. These operators usually refer to global symmetry transforms such as translation or reflection. The corresponding Hamiltonian eigenstates are also eigenstates of the irreducible representations of the symmetry operators, leading to states with definite parity (reflection) [16] or Bloch-states (discrete translation) [17]. For operators $\hat{\Sigma}_L$ describing local symmetries valid only in a limited spatial domain, however, we have $[\hat{H}, \hat{\Sigma}_L] \neq 0$ in general. Does this mean that local symmetries do not affect the eigenstates of the Hamiltonian? Or is it possible to gain additional knowledge about the structure of eigenstates in locally symmetric systems using other means?

Recently a framework for the treatment of local symmetries in one-dimensional discrete setups has been established [18], motivated by corresponding results for discrete local symmetries in continuous one-dimensional systems [19–22]. The very spirit of this framework is to use local symmetries to define new quantities, so-called non-local currents obeying suitably defined continuity equations. For eigenstates in one-dimensional discrete systems, the non-local currents have two interesting properties: Firstly, for a given site the sum of a source term and the two ingoing non-local currents vanishes. This is a Kirchhoff-type law, and because it contains non-local currents, we call it a *non-local* Kirchhoff law. Secondly, the non-local currents are piecewise constant throughout the corresponding domains of local symmetry, no matter how the wavefunction looks like, and thus this constancy may be used to derive a first insight into the structure of the system's eigenstates.

The present paper is mainly motivated by two questions: (i) How can one generalize the above framework to two-dimensional setups and (ii) are there structural properties of eigenstates that may be derived using non-local currents? Considering the first question, two important differences between one and higher discrete dimensions must be considered. The first difference is related to the possible number of different local symmetries. In one dimension there are only reflection and translation symmetries on the line. In higher dimensions, discrete rotations, plane reflections and even more general site permutations are possible. The second difference is related to the possible non-uniform connectivity across a system. That is, sites at different locations may have differing numbers of neighbours. Examples of this kind are Lieb lattices [23,24] or in general all systems where the number of neighbours can vary from site to site. In this work we focus on eigenstates and extend the framework of non-local currents to planar setups. In contrast to one-dimensional systems, the non-local Kirchhoff law for stationary states here generally includes more than two currents for a given site, depending on its connectivity. This means that the non-local currents are no longer constant, even within domains of local symmetries. However, we give conditions that enable one to derive a summed non-local current which is, even in planar systems, constant throughout a domain of local symmetry. Furthermore,