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# Can quantum probes satisfy the weak equivalence principle?

Luigi Seveso [a,*], Matteo G.A. Paris [a,b]

[a] Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy
[b] INFN, Sezione di Milano, I-20133 Milano, Italy

## HIGHLIGHTS

- Can quantum probes under gravity be approximated as test-bodies?
- A formulation of the weak equivalence principle for quantum probes is proposed.
- Quantum probes are found to violate it as a matter of principle.

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ABSTRACT

We address the question whether quantum probes in a gravitational field can be considered as test particles obeying the weak equivalence principle (WEP). A formulation of the WEP is proposed which applies also in the quantum regime, while maintaining the physical content of its classical counterpart. Such formulation requires the introduction of a gravitational field not to modify the Fisher information about the mass of a freely-falling probe, extractable through measurements of its position. We discover that, while in a uniform field quantum probes satisfy our formulation of the WEP exactly, gravity gradients can encode nontrivial information about the particle's mass in its wavefunction, leading to violations of the WEP.



* Corresponding author.
*E-mail address:* luigi.seveso@unimi.it (L. Seveso).

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## 1. Introduction

The weak equivalence principle (WEP) is one of the foundational bedrock of classical gravitational theory [1–3]. It states that the solutions of the equations of motion for a structureless particle falling in a gravitational field exhibit a special form of universality: they do not depend on the particle's mass. Since the mass represents the charge through which the particle couples to gravity, the WEP suggests that gravity may be fundamentally different from the other forces of nature. In fact, the WEP lies at the basis of the possibility of describing gravity in purely geometric terms [4].

However, the test bodies that appear in its formulation are just a classical idealization. Physical particles are consistently described only in a quantum framework. The question therefore arises whether a form of universality (i.e. independence from the probe's internal properties) also holds for quantum particles falling under gravity and, if not, how does the principle of equivalence emerges in the classical limit.

Assessing the validity of the WEP for freely-falling quantum particles offers interesting conceptual challenges [5–10]. In fact, the formulation of the WEP in general relativity refers to test particles with a conserved four-momentum, moving along precise trajectories. However, the description of the dynamics of a quantum particle in terms of a wavefunction is markedly different. First, the wavefunction is not by itself localized, which calls into question the abstraction of a test body in relation to a quantum probe. Second, neither position nor momentum of a propagating wavepacket are well-defined classical variables, but instead represent incompatible observables whose measurements are subject to quantum fluctuations according to the uncertainty principle. Therefore, the Galilean procedure of preparing probes in identical dynamical conditions (same initial position and velocity), and letting them evolve freely, loses operational meaning. The concept of a trajectory dissipates and one can only speak about the results of position measurements. As a consequence, the theory of quantum measurements is expected to play an important part in the formulation of any quantum version of the WEP.

These fundamental difficulties [11] may be ascribed to the fact that a quantum particle does not follow a unique trajectory, making it challenging to associate a unique geometry to spacetime. In fact, from the viewpoint of the path integral formulation of quantum mechanics [12], a particle follows all possible trajectories between two fixed spacetime events. Even if the classical trajectory represents the most important contribution to the total amplitude, fluctuations around it are expected from nearby trajectories. Such fluctuations arise in powers of Planck's constant $\hbar$, so that when $\hbar \to 0$ only the classical trajectory predicted by general relativity survives. The present paper aims to discuss the problem of what becomes of the WEP in the opposite regime, i.e. when quantum fluctuations are turned on.

Apart for the previously mentioned problems, a further difficulty is linked to the fact that the quantum dynamics of a probe under gravity is often mass-dependent [13]. One may have thought of identifying universality of free-fall in the quantum regime with mass-independence of the wavefunction. After all, the wavefunction provides a complete description of the physical state of a quantum system and thus plays a role similar to the solution of the equations of motion in the classical setting. However, the mass of a particle appears explicitly in the dynamical evolution equations—which is in stark contrast with the theory of classical point particles in gravitational fields.

For example, in the non-relativistic limit, the action for a classical particle of mass $m$ in a Newtonian potential $\varphi$ is

$$S = \int dt \left( \frac{m\dot{\mathbf{x}}^2}{2} - m\varphi \right). \tag{1}$$

The mathematical statement of the universality of free-fall is the fact that $m$ appears only as a multiplicative constant, and thus does not enter the equations of motion. The same happens in a fully relativistic (but still classical) context where the action takes the form

$$S = -mc \int ds, \tag{2}$$