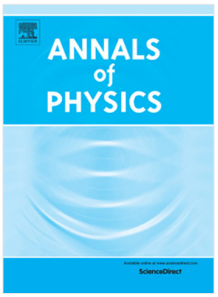

Solutions of the bi-confluent Heun equation in terms of the Hermite functions

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Solutions of the bi-confluent Heun equation in terms of the Hermite functions

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We construct an expansion of the solutions of the bi-confluent Heun equation in terms of the Hermite functions. The series is governed by a three-term recurrence relation between successive coefficients of the expansion. We examine the restrictions that are imposed on the involved parameters in order that the series terminates thus resulting in closed-form finite-sum solutions of the bi-confluent Heun equation. A physical application of the closed-form solutions is discussed. We present the five six-parametric potentials for which the general solution of the one-dimensional Schrödinger equation is written in terms of the bi-confluent Heun functions and further identify a particular conditionally integrable potential for which the involved bi-confluent Heun function admits a four-term finite-sum expansion in terms of the Hermite functions. This is an infinite well defined on a half-axis. We present the explicit solution of the one-dimensional Schrödinger equation for this potential and discuss the bound states supported by the potential. We derive the exact equation for the energy spectrum and construct an accurate approximation for the bound-state energy levels.

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1. Introduction

The bi-confluent Heun equation is widely encountered in contemporary physics and mathematics research [1-5]. For example, in nuclear and atomic physics this equation frequently appears in studying the motion of quantum particles in one-, two- or three-dimensional confinement potentials [1]. The double-well quartic and sextic anharmonic oscillator potentials and the special class of singular confinement potentials consisting of a combination of Coulomb, linear and harmonic potentials are well-known examples of this class of potentials [6,7]. The recent examples include the inverse square root potential [8] and its conditionally exactly integrable generalization [9], applications to quantum chemistry [10], quantum dots [11], and quantum two-state systems [12].

Due to its wide appearance in theoretical physics, mathematical properties of the bi-confluent Heun equation have been studied by many authors (see, e.g., [1-3,13-26]). In particular, the power-series solutions near the regular singularity at the origin and in the neighborhood of the irregular singularity at the infinity [17,18], the continued fraction technique [19] and the Hill determinant approach [20] for a class of confinement potentials