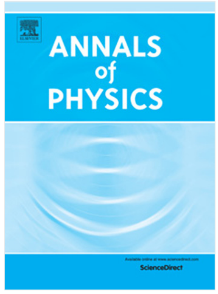
Accepted Manuscript



Superintegrability of the Fock-Darwin system

E. Drigho-Filho, Ş. Kuru, J. Negro, L.M. Nieto

PII: S0003-4916(17)30132-X
DOI: http://dx.doi.org/10.1016/j.aop.2017.05.003
Reference: YAPHY 67386

To appear in: *Annals of Physics*

Received date : 22 March 2017
Accepted date : 7 May 2017

Superintegrability of the Fock-Darwin system

E. Drigho-Filho[a], Ş. Kuru[b], J. Negro[c] and L.M. Nieto[c]

[a]Departamento de Fisica, Universidade Estadual Paulista-UNESP,
15054-000 S. Jose do Rio Preto, SP, Brazil
[b]Department of Physics, Faculty of Science, Ankara University, 06100 Ankara, Turkey
[c]Departamento de Física Teórica, Atómica y Óptica, Universidad de Valladolid,
47011 Valladolid, Spain

May 9, 2017

Abstract

The Fock-Darwin system is analysed from the point of view of its symmetry properties in the quantum and classical frameworks. The quantum Fock-Darwin system is known to have two sets of ladder operators, a fact which guarantees its solvability. We show that for rational values of the quotient of two relevant frequencies, this system is superintegrable, the quantum symmetries being responsible for the degeneracy of the energy levels. These symmetries are of higher order and close a polynomial algebra. In the classical case, the ladder operators are replaced by ladder functions and the symmetries by constants of motion. We also prove that the rational classical system is superintegrable and its trajectories are closed. The constants of motion are also generators of symmetry transformations in the phase space that have been integrated for some special cases. These transformations connect different trajectories with the same energy. The coherent states of the quantum superintegrable system are found and they reproduce the closed trajectories of the classical one.

PACS: 03.65.-w 02.30.Ik

KEYWORDS: Fock-Darwin system, quantum dot, superintegrability, factorization, higher-order symmetry, coherent state.

1 Introduction

In this work, we will revisit the Fock-Darwin (FD) system [1, 2] with two main purposes: to examine in close detail its symmetries and its superintegrability character, and to give a complete picture of the system in both, the quantum and classical frameworks. The FD system consists in a charged particle moving in the plane and confined by a harmonic potential under an external uniform magnetic field. Here, we are not taking into account the spin splitting in the magnetic field since this can be directly added at any stage.

The FD system has a number of applications in several fields. For example, it is used as frequent ingredient of quantum dots. Due to the small size (of a few nanometers), when the discrete energy levels are filled with electrons, the quantum dot is called artificial atom, an entity whose properties have been recently described. If there are more than one electron confined in the quantum dot, the Coulomb interaction has to be taken into account. In this case, approximation methods, like diagonalization of the Hamiltonian matrix or the constant interaction model [3, 4, 5, 6, 7, 8], are available.

In works dealing with quantum dots, the connection between 'accidental degeneracy' and the symmetry group of a Hamiltonian has attracted considerable attention [9]. This connection