Annals of Physics 383 (2017) 130–139

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Phase space formulation of radiative transfer in optically thick plasmas

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ARTICLE INFO

Article history:
Received 2 March 2017
Accepted 13 May 2017
Available online 16 May 2017

Keywords:
Radiative transfer
Quantum phase space
Wigner function
Plasma spectroscopy

ABSTRACT

We present a formulation of the radiative transfer theory based on the quantum phase space formalism. The formalism employs the Wigner function relative to the electric field in $(\mathbf{r}, t, \mathbf{k}, \omega)$ space. It is shown that this quantity obeys a transport equation with source and loss terms nonlocal in space and time. This delocalization is a feature of the Heisenberg uncertainty relation, both relative to position and momentum and to time and energy. A discussion of the theory, together with links to the standard radiative transfer formalism, is done.

1. Introduction

The radiative transfer theory serves as a framework for the description of opacity effects in plasma spectroscopy; applications include astrophysics [1], inertial and magnetic fusion [2–8], engineering (lamps, lasers) [9,10], and also biomedical optics [11]. It involves a transport equation for the radiation specific intensity, which is established phenomenologically from energy balance considerations. We reconsider this theory from a first principles approach based on quantum electrodynamics. Recently [12–14], a transport model for light has been derived in the framework of the quantum phase space formalism introduced by Wigner [15]. This approach employs the photon Wigner function as a basic quantity of interest and it yields a transport equation with source and loss terms nonlocal in the phase space, which captures features of the Heisenberg uncertainty relation. An analysis of this equation and numerical applications have indicated the possibility of an alteration of the photon emission and absorption processes if the radiation is partially coherent, viz., in the case of narrow-band radiation like in atomic transitions. We present here an extension of the formalism devoted to clarify the links between the quantum phase space description of radiation and quantities accessible to experiments. Our study is based on the modeling of the electric field autocorrelation function,

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http://dx.doi.org/10.1016/j.aop.2017.05.010
0003-4916/

a quantity which is involved in photodetection signals [16–18]. In strict sense, this function is not directly accessible to the formalism employed in [12–14] because the latter relies on a Wigner function in six-dimensional (position–momentum) phase space, with no explicit mention to the temporal correlation properties of the electric field. We show here that a photodetection signal can be described in terms of a generalization of the Wigner function to space and time coordinates. A transport equation for this quantity is derived in the case of atomic line radiation. The derivation involves the quantized form of the Maxwell wave equation in a dielectric medium. The absorption, stimulated and spontaneous emission processes are accounted for through a closure relation for the polarization density, obtained using a linear response model. The paper is organized as follows: Section 2 gives a summary of formulas obtained within the phenomenological radiative transfer formalism, along the lines of textbooks on spectroscopy; the space–time Wigner function is introduced in Section 3; a transport equation for this quantity is derived and discussed in Section 4; finally, the problem of radiation absorption in an optically thick medium is addressed in Section 5, in the framework of plasma spectroscopy applications.

2. Radiative transfer formalism: a summary of formulas

The radiative transfer formalism presented in textbooks (e.g. [1]) is usually introduced in a phenomenological way. The specific intensity $I(\mathbf{r}, t, \mathbf{n}, \nu)$, which serves as a fundamental quantity of interest, is defined with geometric and energetic arguments. Consider a radiation pencil with spectral band $[\nu, \nu+d\nu]$ propagating along the direction $\mathbf{n}$ ($\mathbf{n}$ is a unitary vector) and crossing a surface element dS located at $\mathbf{r}$; by definition, the specific intensity is such that the energy transported by radiation between times t and $t + dt$ is $\delta E = I(\mathbf{r}, t, \mathbf{n}, \nu)\cos\theta d\nu d\Omega dS dt$, where the element $d\Omega$ denotes the angular aperture and θ is the angle between the radiation pencil and the normal to the surface. It is customary to associate a wave packet (referred to as 'photon') particle distribution function with the specific intensity through the proportionality relation $I(\mathbf{r}, t, \mathbf{n}, \nu) = (h^4\nu^3/c^2)f(\mathbf{r}, \mathbf{p}, t)$, with the correspondence $\mathbf{p} = h\nu\mathbf{n}/c$. In a dispersive medium, the substitution $c \to c/n$ is done where n is the refractive index, e.g. [19]. The distribution function is normalized in such a way that $\int d^3r \int d^3p f(\mathbf{r}, \mathbf{p}, t)$ is identical to the total number of photons present in the medium; the momentum volume element d^3p is defined as $|\mathbf{p}|^2 d|\mathbf{p}| d\Omega$, which provides a link to the element $d\nu d\Omega$ involved in the specific intensity definition. A transport equation for the specific intensity (radiative transfer equation) is obtained from energetic balance considerations. When applied to the radiation due to dipoles, this equation reads

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\boldsymbol{\nabla}\right) I = \eta - \kappa I, \tag{1}$$

and, again, substitutions involving the refractive index n are done if the medium is dispersive [19]. The source and loss terms (emission and extinction coefficients) involve atomic parameters:

$$\eta(\mathbf{r}, t, \mathbf{n}, \nu) = \frac{h\nu_{ul}}{4\pi} N_u(\mathbf{r}, t) A_{ul} \psi(\mathbf{r}, t, \mathbf{n}, \nu), \tag{2}$$

$$\kappa(\mathbf{r}, t, \mathbf{n}, \nu) = \frac{h\nu_{ul}}{4\pi} [N_l(\mathbf{r}, t) B_{lu} \phi(\mathbf{r}, t, \mathbf{n}, \nu) - N_u(\mathbf{r}, t) B_{ul} \psi(\mathbf{r}, t, \mathbf{n}, \nu)]. \tag{3}$$

The A and B coefficients are the Einstein coefficients for spontaneous emission, absorption and stimulated emission; N_u and N_l stand for the densities of atoms in upper and lower levels of the transition; ν_{ul} is the Bohr frequency of the transition $u \to l$ under consideration; ψ and ϕ are frequency-normalized emission and absorption line shape functions, viz. $\int (d\Omega/4\pi) \int d\nu \psi(\mathbf{r}, t, \mathbf{n}, \nu) = \int (d\Omega/4\pi) \int d\nu \phi(\mathbf{r}, t, \mathbf{n}, \nu) = 1$. The line shapes are proportional to the Fourier transform of the atomic dipole autocorrelation function; they characterize the energy level perturbation during an emission or an absorption process. The emission and absorption profiles are identical if the emission and absorption processes are statistically independent (complete redistribution). In a more general case, they are related through an integral relation that involves a two-photon line shape