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Is zero-point energy physical? A toy model for Casimir-like effect

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HIGHLIGHTS

- A toy model for Casimir-like effect with only 3 degrees of freedom is constructed.
- Casimir vacuum can be related to the photon vacuum by a non-trivial Bogoliubov transformation.
- Casimir vacuum is a ground state only for an effective Hamiltonian describing Casimir plates at a fixed distance.
- At the fundamental microscopic level, Casimir force is best viewed as a manifestation of van der Waals forces.

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ABSTRACT

Zero-point energy is generally known to be unphysical. Casimir effect, however, is often presented as a counterexample, giving rise to a conceptual confusion. To resolve the confusion we study foundational aspects of Casimir effect at a qualitative level, but also at a quantitative level within a simple toy model with only 3 degrees of freedom. In particular, we point out that Casimir vacuum is not a state without photons, and not a ground state for a Hamiltonian that can describe Casimir force. Instead, Casimir vacuum can be related to the photon vacuum by a non-trivial Bogoliubov transformation, and it is a ground state only for an effective Hamiltonian describing Casimir plates at a fixed distance. At the fundamental microscopic level, Casimir force is best viewed as a manifestation of van der Waals forces.

So, in the discussion session after Casimir's lecture I switched topic and asked: "Is the Casimir effect due to the quantum fluctuations of the electromagnetic field, or is it due to the van der Waals forces between the molecules in the two media?" Casimir's answer began, "I have not made up my mind". (I.H. Brevik, from the Foreword in [1].)

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1. Introduction

In physics we measure energy differences, not absolute energies. Zero-point energy, that is energy of the ground state, is therefore unphysical and can be removed by a simple subtraction. Yet Casimir effect [2], in its simplest form described as an attractive force between electrically neutral plates, is often presented as a demonstration that zero-point energy is physical [3–10]. On the other hand, Casimir effect can also be explained as a force that originates from van der Waals forces [1,11–18], without referring to zero-point energy. There are arguments that the explanation in terms of van der Waals forces is more fundamental [19,20] (see also [1,17]), yet some consider the question of the true nature of Casimir effect as a controversy [21–23] that still needs to be resolved.

A part of the difficulty stems from the fact that explanation of Casimir effect requires quantum electrodynamics (QED), which involves various technical difficulties coming from the fact that QED is a quantum field theory, i.e. a theory with an infinite number of degrees of freedom. To overcome this technical difficulty, in this paper we shall not be so much concerned with technical details of Casimir effect itself. Instead, our main goal will be to understand in detail how a Casimir-like effect can emerge *in general*, using only general properties of quantum mechanics. For that purpose we shall study a toy model with only 3 degrees of freedom, which under certain approximations can be reduced to 2 or even 1 degree of freedom. The toy model will be chosen such that it has many conceptual similarities with the real Casimir effect, but is technically much simpler than that. This will enable us to understand relatively easily where the effect comes from, and how is it related to the zero-point energy. In addition, to make a contact with actual Casimir physics, we shall discuss various aspects of Casimir effect at a qualitative non-technical level.

The paper is organized as follows. In Section 2 we study various conceptual aspects of Casimir effect. In particular, we explain the difference between ground state and vacuum, point out that Casimir force can be attributed to the interacting vacuum energy but not to the ground state energy, discuss the role of dielectric constant and its relation to van der Waals force, and make a motivation for the toy model discussed in the following sections. In Section 3 we introduce our toy model and analyze its classical properties. In Section 4 we explain how the toy model leads to a quantum Casimir-like force, using both a Casimir-like and a Lifshitz-like approach. In Section 5 we explore the content of the interacting vacuum by using the Bogoliubov-transformation method. In Section 6 we discuss how the calculations in our toy model are related to the calculations for real Casimir effect. Finally, the conclusions are drawn in Section 7.

As a disclaimer, we also want to remark that in this paper we do not study the relevance of ground-state energy to gravitational physics. For a possible relevance of Casimir energy to gravitational phenomena see e.g. [24] and references therein.

2. Basic conceptual questions

One of the main messages of this paper, in agreement with [19,20], is that, at the fundamental microscopic level, Casimir effect should be viewed as a manifestation of van der Waals forces, and not as a manifestation of zero-point energy. But why then the effect is often attributed to zero-point energy and why such a description works fine too? And what exactly are drawbacks of the zero-point energy description? In this section we give a non-technical answer to those and many other related conceptual questions.

2.1. What is vacuum?

In physics, the word "vacuum" has many different meanings. It can mean a state without any particles whatsoever, or a state without only one kind of particles such as photons, or a state annihilated by some lowering operators, or a local minimum of a classical potential, or the state with the lowest possible energy. Of course, all these notions of "vacuum" are closely related, but the point is that they are not strictly identical.

Which of those notions of "vacuum" is relevant for the description of Casimir effect? Clearly, Casimir vacuum is not a state without any particles whatsoever, because Casimir effect involves plates made of atoms. As we shall see, Casimir vacuum is also not a local minimum of a classical potential, and perhaps more surprisingly, not even a state without photons. We shall see that Casimir vacuum is a state annihilated by some lowering operators which are *not photon* lowering operators.