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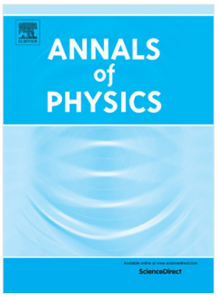

Asymptotic behaviour of time averages for non-ergodic Gaussian processes

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Asymptotic behaviour of time averages for non-ergodic Gaussian processes

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Abstract

In this work, we study the behaviour of time-averages for stationary (non-ageing), but ergodicity-breaking Gaussian processes using their representation in Fourier space. We provide explicit formulas for various time-averaged quantities, such as mean square displacement, density, and analyse the behaviour of time-averaged characteristic function, which gives insight into rich memory structure of the studied processes. Moreover, we show applications of the ergodic criteria in Fourier space, determining the ergodicity of the generalised Langevin equation's solutions.

Keywords: ergodicity breaking, Gaussian process, statistical analysis, generalised Langevin equation

1. Introduction

1.1. The goal

The relation between the time averages and ensemble averages is one of the most important topics of statistical physics and this area of research is under intense development. The abstract, mathematical ergodic theory has become a very wide subject, but in the recent years a new trend has emerged which concentrates on very practical questions.

One example is the behaviour of the time-averaged mean square displacement

$$\overline{\delta^2}(\Delta) := \frac{1}{T-\Delta}\int_0^{T-\Delta} \mathrm{d}\tau \, \big(X(\tau+\Delta)-X(\tau)\big)^2. \tag{1}$$

This quantity can be estimated using only one (sufficiently long) trajectory in contrast to the ensemble-averaged mean square displacement

$$\delta^2(\Delta) := \int \mathrm{dP} \, \big(X(t+\Delta)-X(t)\big)^2, \tag{2}$$

which requires many trajectories to estimate. The comparison of these two types of mean square displacement is central in study of the weak ergodicity