Annals of Physics 383 (2017) 357–388

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

# Dynamics of self-interacting strings and energy-momentum conservation

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## HIGHLIGHTS

- Ultraviolet singularities jeopardize the dynamics of classical superstring theory.
- A new fundamental principle to construct a well-defined dynamics is introduced.
- The method is based on a distribution-valued energy-momentum tensor.
- The principle establishes in a constructive way a finite self-force of a classical string.
- The proposed mechanism of cancelation of ultraviolet divergences is universal.

ARTICLE INFO

*Article history:*
Received 20 October 2016
Accepted 30 May 2017
Available online 15 June 2017

*Keywords:*
Classical string theory
Self-interaction
Renormalization
Energy-momentum conservation
Distribution theory

ABSTRACT

Classical strings coupled to a metric, a dilaton and an axion, as conceived by superstring theory, suffer from ultraviolet divergences due to self-interactions. Consequently, as in the case of radiating charged particles, the corresponding effective string dynamics cannot be derived from an action principle. We propose a *fundamental principle* to build this dynamics, based on local energy-momentum conservation in terms of a well-defined distribution-valued energy-momentum tensor. Its continuity equation implies a finite equation of motion for self-interacting strings. The construction is carried out explicitly for strings in uniform motion in arbitrary space–time dimensions, where we establish cancelations of ultraviolet divergences which parallel superstring non-renormalization theorems. The uniqueness properties of the resulting dynamics are analyzed.


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http://dx.doi.org/10.1016/j.aop.2017.05.024

## 1. Introduction

In the same way as charged particles in four space–time dimensions are subject to divergent electromagnetic self-interactions, generic charged extended objects, $p$-branes, in $D$ space–time dimensions are subject to infinite self-interactions. The reason for this is that the fields created by a brane become singular on the brane world-volume, meaning that the *self-fields*, and hence the *self-forces*, are infinite. A – in a certain sense dramatic – consequence of these ultraviolet divergences is that the theory of self-interacting branes cannot be derived from a variational principle: while the original fundamental equations of motion for fields and branes follow of course from an action principle, once one substitutes the fields resolving the formers in the equations of motion of the latter, the resulting equations are divergent. If one isolates and subtracts – adapting whatever prescription – the infinities, the resulting non-local equations of motion of the brane do no longer follow from an action principle. This in turn implies that the conservation laws, in particular energy–momentum conservation, cannot be derived from Noether's theorem, see *e.g.* [1–3] for the case of self-interacting charged particles and dyons in $D = 4$. Within this approach one looses thus the control over energy-momentum conservation.

More precisely ultraviolet divergences show up in brane theory in two, a priori, unrelated physical quantities: (i) in the *self-force* of the brane, *i.e.* the force exerted by the field generated by the brane on the brane itself, as explained above, and (ii) in the $D$-momentum contained in a volume $V$ enclosing (a portion of) the brane. Although the origins of the divergences appearing in these two quantities – the self-force and the $D$-momentum – are the same, *i.e.* the bad ultraviolet behavior of the field in the vicinity of the brane, their cures require actually two distinct unrelated procedures [4].

To cure the divergent self-force one may proceed, as anticipated above, regularizing the field produced by the brane in some way, evaluating it on the brane and trying then to isolate and subtract the divergent terms.

The cure of the infinite $D$-momentum requires instead the construction of a well-defined *distribution-valued* energy-momentum tensor and offers – at the same time – a strategy for the derivation of the self-force, that is alternative to the approach described above and overcomes its main drawback, *i.e.* the missing control over energy-momentum conservation. It works as follows.

Generically the standard total energy-momentum tensor has the structure

$$\tau^{\mu\nu} = \tau^{\mu\nu}_{\text{field}} + \tau^{\mu\nu}_{\text{kin}}, \quad \tau^{\mu\nu}_{\text{kin}} = M \int l^{\mu\nu} \delta^D(x - y(\sigma))\sqrt{\gamma}\, d^2\sigma, \tag{1.1}$$

where $\tau^{\mu\nu}_{\text{kin}}$ is the free *kinetic* energy-momentum tensor of the brane (with $M$ the brane tension and $y^\mu(\sigma)$ the brane coordinates, see Sections 2 and 3.1 for the notations) and $\tau^{\mu\nu}_{\text{field}}$ is the *bare* energy-momentum tensor produced by the fields[1] : while the fields – solutions of linear d'Alembert equations – are by definition distributions, the tensor $\tau^{\mu\nu}_{\text{field}}$ – a product of the fields – is *not* a distribution. Consequently, (i) the $D$-momentum of the field

$$P^\mu_V = \int_V \tau^{0\mu}_{\text{field}}\, d^3x$$

contained in a volume $V$ is in general divergent and, (ii) it makes no sense to evaluate the divergence $\partial_\mu \tau^{\mu\nu}_{\text{field}}$ to analyze the conservation properties of $\tau^{\mu\nu}$. The cure of these pathologies requires the construction of a *renormalized* distribution-valued energy-momentum tensor $T^{\mu\nu}_{\text{field}}$, out of $\tau^{\mu\nu}_{\text{field}}$. A – in principle standard – way to do this consists in the introduction of a regularization – preserving possibly Lorentz- as well as reparameterization-invariance – and the subsequent subtraction from the regularized energy-momentum tensor $(\tau^{\mu\nu}_{\text{field}})_{reg}$ of *divergent local counterterms*, *i.e.* of counterterms supported on the brane that do not converge to distributions as the regularization is removed. By

1 Actually in a generic brane- or string-model, as the one considered in this paper, this tensor is given by a sum $\tau^{\mu\nu}_{\text{field}} = \tau^{\mu\nu}_f + \tau^{\mu\nu}_{int}$, where $\tau^{\mu\nu}_f$ depends only on the fields and is supported on the bulk, and $\tau^{\mu\nu}_{int}$ is a field-brane interaction-term supported on the world-volume.