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Quantum walks and gravitational waves

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HIGHLIGHTS

- Explicit construction of quantum walks coupled to $(1 + 2)$D gravitational fields.
- Study of $(1 + 2)$D quantum walks coupled to gravitational waves.
- Interference patterns of quantum walks in gravitational waves.

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ABSTRACT

A new family of discrete-time quantum walks (DTQWs) propagating on a regular $(1 + 2)$D spacetime lattice is introduced. The continuum limit of these DTQWs is shown to coincide with the dynamics of a Dirac fermion coupled to an arbitrary relativistic gravitational field. This family is used to model the influence of arbitrary linear gravitational waves (GWs) on DTQWs. Pure shear GWs are studied in detail. We show that on large spatial scales, the spatial deformation generated by the wave induces a rescaling of the eigen-energies by a certain anisotropic factor which can be computed exactly. The effect of pure shear GWs on fermion interference patterns is also investigated, both on large scales and on scales comparable to the lattice spacing.

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1. Introduction

Discrete-time quantum walks (DTQWs) are unitary quantum automata. They are not stochastic but can be viewed nevertheless as formal quantum analogues of classical random walks. They were first introduced by Feynman [1,2] in the 1940s and later re-introduced, as 'quantum random walks' by Aharonov et al. [3], and in a systematic way by Meyer [4]. DTQWs have been realized experimentally with a wide range of physical objects and setups [5–11], and are studied in a large variety of contexts, ranging from fundamental quantum physics [6,12] to quantum algorithmics [13,14], solid-state physics [15–18] and biophysics [19,20].

It has been shown recently that the continuum limit of several DTQWs defined on regular $(1 + 1)$D spacetime lattices coincides with the dynamics of Dirac fermions coupled, not only to electric fields [21,22], but also to arbitrary non-Abelian Yang–Mills gauge fields [23] and to relativistic gravitational fields [24–27]. Gravitational waves (GWs) [28] are of great interest, both theoretically and experimentally, and their effects on DTQWs are thus worth investigating. The interest about GWs has been renewed by their recent direct detection [29]. Linear GWs resemble electromagnetic waves. In particular, GWs can be expanded as a superposition of plane waves and each plane wave as a superposition of two polarization states, both polarizations being perpendicular to the direction of propagation. The effect of these plane GWs on matter is thus typically studied in the polarization plane and it makes little sense to envisage the action of GWs on $(1 + 1)$D DTQWs. Performing a valid study of how GWs influence DTQWs thus requires building DTQWs coupled to $(1+2)$D gravitational fields.

We start by presenting a new family of $(1 + 2)$D DTQWs whose continuum limit coincides with the dynamics of a Dirac fermion coupled to arbitrary $(1+2)$D gravitational fields. The construction of this family is inspired by the $(1 + 1)$D construction presented in [25]. The DTQWs in the $(1 + 2)$D spacetime depend on two parameters, which code for the mass of the walker and for the finite spacing of the lattice, and on four time- and space-dependent angles. We then show how to choose these four angles to describe linear GWs. A generic linear GW on the lattice can be considered as the superposition of three waves: two compression waves along the directions of the lattice and a shear wave coupling directly two directions of the lattice through non-diagonal metric components. Shear effects are of particular interest in relativistic gravitation and are present in generic solutions of Einstein equations [30]. We thus focus on pure shear GWs and examine in detail their action on the DTQWs. Our main results are the following. On large spatial scales, pure shear GWs rescale locally the eigen-energies by an anisotropic factor, to make up for the space deformation induced by the wave, and the eigen-polarizations are modified as well. On smaller scales comparable to a few lattice spacings, both polarizations and energies are modified in a non-trivial way; this has the effect of changing significantly the interference pattern of two fermion eigen-modes. A final section discusses the construction of the DTQWs and mentions several avenues open to further study.

2. DTQWs in $(1 + 2)$ dimensions

Consider a quantum walker moving on a two-dimensional lattice (discrete space) with nodes labeled by $(p_1, p_2) \in \mathbb{Z}^2$. Let $j \in \mathbb{N}$ label discrete time and (b_-, b_+) be a certain time- and position-independent basis of the two-dimensional coin Hilbert space of the walker, that we identify to $((1, 0)^\top, (0, 1)^\top)$, where $\top$ denotes the transposition. The state of the walker at time j and point (p_1, p_2) is described by a two-component wave function $\Psi_{j,p_1,p_2} = \psi^-_{j,p_1,p_2} b_- + \psi^+_{j,p_1,p_2} b_+$. The collection $(\Psi_{j,p_1,p_2})_{(p_1,p_2)\in\mathbb{Z}^2}$ will be denoted by Ψ_j.

The time evolution of Ψ_j is fixed by a time-dependent unitary operator V_j:

$$\Psi_{j+1} = V_j \Psi_j. \tag{1}$$

This unitary operator involves two real positive parameters, ϵ and m, and four time- and space-dependent angles θ^{11}, θ^{12}, θ^{21} and θ^{22}; it consists essentially in a combination of rotations in spin space and of translations in physical space, along the two directions of the lattice. The operator V_j consists in a rather complicated combination of rotations in spin space and of translations in physical