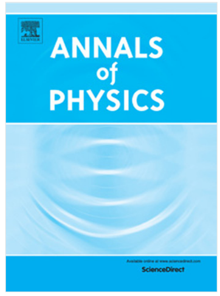

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A REPRESENTATION OF WEYL-HEISENBERG LIE ALGEBRA IN THE QUATERNIONIC SETTING

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Abstract. Using a left multiplication defined on a right quaternionic Hilbert space, linear self-adjoint momentum operators on a right quaternionic Hilbert space are defined in complete analogy with their complex counterpart. With the aid of the so-obtained position and momentum operators, we study the Heisenberg uncertainty principle on the whole set of quaternions and on a quaternionic slice, namely on a copy of the complex plane inside the quaternions. For the quaternionic harmonic oscillator, the uncertainty relation is shown to saturate on a neighborhood of the origin in the case we consider the whole set of quaternions, while it is saturated on the whole slice in the case we take the slice-wise approach. In analogy with the complex Weyl-Heisenberg Lie algebra, Lie algebraic structures are developed for the quaternionic case. Finally, we introduce a quaternionic displacement operator which is square integrable, irreducible and unitary, and we study its properties.

1. Introduction

In the past few years there has been a resurgence of interest for the quaternionic quantum mechanics. This topic was extensively studied starting from the celebrated paper of Birkhoff and von Neumann who asserted that quantum mechanics can be studied only over the complex and the quaternionic numbers, see [11]. It culminated with the book of Adler in 1995 and, after that, the interest on this topic started fading, indeed some crucial ingredients of the theory were missing and prevented further developments of the subject.

As it is well known, quaternions can always be represented as a pair of complex numbers (the so-called symplectic components) and hence quaternions possess a symplectic structure. As in the complex quantum mechanics, states of quaternionic quantum mechanics are represented by vectors of a separable quaternionic Hilbert space and observables are represented by quaternionic linear and self-adjoint operators. However, quaternionic quantum mechanics is different from the complex quantum mechanics [1].

There are three main issues which prevented physicists and mathematicians to construct a well-formed quaternionic quantum mechanics. In this introduction we will discuss them and we will show that, while the answer to one of them was found in the past

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