Accepted Manuscript

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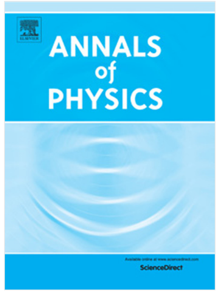

PII: S0003-4916(17)30235-X
DOI: http://dx.doi.org/10.1016/j.aop.2017.08.015
Reference: YAPHY 67467

To appear in: *Annals of Physics*

Received date : 6 February 2017
Accepted date : 18 August 2017

Please cite this article as: T. Qureshi, M.A. Siddiqui, Wave-particle duality in N-path interference, *Annals of Physics* (2017), http://dx.doi.org/10.1016/j.aop.2017.08.015

Wave-Particle Duality in N-Path Interference

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Abstract

Bohr's principle of complementarity, in the context of a two-slit interference experiment, is understood as the quantitative measures of wave and particle natures following a duality relation $\mathcal{D}^2 + \mathcal{V}^2 \leq 1$. Here $\mathcal{D}$ is a measure of distinguishability of the two paths, and $\mathcal{V}$ is the visibility of interference. It is shown that such a relation can be formulated for $N-$slit or $N-$path interference too, with the proviso that the wave nature is characterized by a measure of *coherence* (C). This new relation, $\mathcal{D}^2 + C^2 \leq 1$ is shown to be tight, and reduces to the known duality relation for the case $N = 2$. A recently introduced similar relation [Bagan et al., *Phys. Rev. Lett.* 116, 160406 (2016)] is shown to be inadequate for the purpose.

Keywords: Complementarity, Wave-particle duality, Multi-path interference.
PACS: 03.65.Ta

1. Introduction

Niels Bohr had argued that the wave and particle natures of quantum objects are complementary [1]. An experiment that clearly illustrates one nature, will necessarily obscure the other. Quantum objects, which show the properties of both a particle and a wave are often called quantons [2, 3]. In a two-slit interference experiment, the particle nature is characterized by the ability to tell, which of the two slits the quanton went through. The wave nature, on the other hand, has traditionally been characterized by the interference patterns built up by the successive registering of individual quantons on a screen. The visibility of the interference pattern is a good measure of the wave nature.

An inequality was derived by Englert [4], which can be understood as a quantitative statement of Bohr's complementarity principle

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1, \tag{1}$$

where $\mathcal{D}$ is a *path distinguishability* and $\mathcal{V}$ is the visibility of the interference pattern. One should note that $\mathcal{D}$ here does not itself have a physical meaning, although it is a good measure of the ability to distinguish between the two paths of a quanton. This work built up on various earlier attempts [5, 6, 7]. This duality is now well established, and has also been connected to entropic uncertainty relation [8] and to a dichotomy between symmetry and asymmetry [9], among others.

A natural question then arises, that since wave-particle duality should also hold for multi-slit interference experiments, can a duality relation be formulated for such experiments? Several attempts have been made in this direction, particularly for three-slit interference experiments [10, 11, 12, 13, 14, 15]. Earlier we derived a duality relation for three-slit interference [16], by introducing a new distinguishability $\mathcal{D}_Q$ based on unambiguous quantum state discrimination (UQSD) [17, 18, 19, 20, 21].

For N-path interference a duality relation $\mathcal{D}_Q + C \leq 1$ was derived by Bera et al. [22], where $\mathcal{D}_Q$ is a new distinguishability based on UQSD, and C is a *coherence* based on a recently introduced measure of quantum coherence [23]. This relation has been shown to be tight, and also reduces to Eqn (1) for $N = 2$. Inspite of this, there seems to be an interest in deriving a relation for N-path interference, which has a form similar to (1) [24, 25].

In this paper we formulate a duality relation,

$$\mathcal{D}^2 + C^2 \leq 1, \tag{2}$$

which is in the form of (1), and holds for N-path interference. For the case $N = 2$, it reduces to (1), and the distinguishability $\mathcal{D}$ and coherence C reduce to Englert's distinguishability and visibility [4], respectively.

2. N-path interference

We start by considering a N-path interference experiment. There are N paths available for the quanton to pass through, before it encounters a screen or detector to give rise to interference. We consider a general scenario where the probabilities to pass through different paths (or slits) may be unequal. The state of the quanton after passing through the N paths may be written as $|\Psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle + c_3|\psi_3\rangle + \cdots + c_N|\psi_N\rangle$, where $|\psi_i\rangle$ is the possible state of the quanton if it passes through the i'th path (or slit), and c_i is the amplitude for taking that path. If $\{|\psi_i\rangle\}$ are orthonormal, c_i should satisfy $\sum_i |c_i|^2 = 1$.

Consider now a path-detector which is capable of recording which path the quanton followed. This path detector is also a quantum object. According to von Neumann's criteria of a

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