Annals of Physics 384 (2017) 61–70

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Magneto-optical conductivity of anisotropic two-dimensional Dirac–Weyl materials

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H I G H L I G H T S

- The magneto-optical response of anisotropic massless Dirac fermions is investigated.
- The conductivity tensor is obtained from the Kubo formula.
- We study effects of strain and magnetic field on the light absorption of graphene.
- The Faraday rotation in strained graphene is analytically determined.

A R T I C L E I N F O

Article history:
Received 25 May 2017
Accepted 20 June 2017
Available online 8 July 2017

Keywords:
Anisotropic Dirac fermions
Strained graphene
Landau levels
Magneto-optical conductivity
Faraday effect

A B S T R A C T

In the presence of an external magnetic field, the optical response of two-dimensional materials, whose charge carriers behave as massless Dirac fermions with arbitrary anisotropic Fermi velocity, is investigated. Using Kubo formalism, we obtain the magneto-optical conductivity tensor for these materials, which allows to address the magneto-optical response of anisotropic Dirac fermions from the well known magneto-optical conductivity of isotropic Dirac fermions. As an application, we analyse the combined effects of strain-induced anisotropy and magnetic field on the transmittance, as well as on the Faraday rotation, of linearly polarized light after passing strained graphene. The reported analytical expressions can be a useful tool to predict the absorption and the Faraday angle of strained graphene under magnetic field. Finally, our study is extended to anisotropic two-dimensional materials with Dirac fermions of arbitrary pseudospin.

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http://dx.doi.org/10.1016/j.aop.2017.06.013

1. Introduction

A Dirac–Weyl material, such as graphene [1,2], organic conductors [3,4] and topological insulators [5,6], possesses low-energy fermionic excitations that behave as massless Dirac particles, rather than conventional fermions governed by the Schrödinger's equation [7]. The behaviour of these Dirac fermions in graphene has been studied by applying an external magnetic field, where a half-integer quantum Hall effect was observed [1,2]. This observation demonstrates the existence of relativistic Landau levels with a square root dependence on both the magnetic field B and Landau level index n (as $\sim \sqrt{B|n|}$), which is in stark contrast to the equally spaced Landau levels for a conventional two-dimensional electron gas. This unconventional Landau spectrum has also been proved by means of infrared spectroscopy measurements, whose transmittance through graphene under magnetic field is in excellent agreement with the theoretical magneto-optical response of Dirac fermions derived from the Kubo formula [8,9]. Moreover, graphene exhibits quantum Faraday and Kerr rotations associated with the half-integer quantum Hall effect [10,11].

Even in absence of magnetic field the optical properties of graphene are *per se* unusual. For example, graphene presents an universal transmittance T determined by the fine-structure constant α (being $T \approx 1 - \pi\alpha \approx 97.7\%$), over a broad range of frequencies [12]. This remarkable feature is a consequence of its charge carriers behaved as massless Dirac fermions. At the same time, graphene exhibits a large interval of elastic response and then, mechanical deformations have been proposed as a tool to tune its optical properties [13–16]. By applying a uniaxial strain, the optical conductivity of graphene becomes anisotropic [17,18] and its transmittance depends on the incident light polarization [14,19].

Up to now, the combined effects of both magnetic field and strain on the optical properties of a two-dimensional Dirac–Weyl material (2D DWM) have not been analysed in detail. In fact, the optical conductivity of unstrained graphene under magnetic field is given by an antisymmetric tensor [20], whereas the optical response of strained graphene, in absence of magnetic field, is a symmetric tensor [21]. In consequence: *What is the symmetry of the optical conductivity tensor if both effects are present? How many independent components does this tensor have?*

The main objective of this article is to provide a general formulation of the magneto-optical conductivity for anisotropic (strained) 2D DWMs. For this purpose, in Section 2 we start by deriving the Landau level spectrum for the mentioned 2D DWMs. Unlike previous approaches [22,23], our derivations are carried out in an arbitrary laboratory reference system. In Section 3, we give an analytical expression for the magneto-optical conductivity tensor of an anisotropic 2D DWM, while we answer the above questions in Section 4. As an example, we apply our analytical results to a strained graphene and we report a generalized Faraday rotation. Section 6 is devoted to discuss the extension of this analysis to Dirac fermions with arbitrary pseudospin and, finally, some conclusions are given in Section 7.

2. Landau levels

We consider the dynamics of low-energy carriers in an anisotropic 2D DWM governed by the generic Dirac–Weyl Hamiltonian [24–28]

$$\mathcal{H} = \boldsymbol{\tau} \cdot \mathbf{v} \cdot \boldsymbol{p} = \sum_{i,j} \tau_i v_{ij} p_j, \tag{1}$$

where $\boldsymbol{\tau} = (\tau_x, \tau_y)$ are the first two Pauli matrices that act on the pseudospin degree of freedom, $\boldsymbol{p}$ is the momentum measured from the Dirac point and $\mathbf{v}$ is the (2×2) symmetric Fermi velocity tensor. The corresponding energy dispersion relation is

$$E(\boldsymbol{p}) = \pm\sqrt{\sum_i \Big(\sum_j v_{ij} p_j\Big)^2}, \tag{2}$$

which represents elliptic Dirac cones. Unlike an isotropic 2D DWM, whose Hamiltonian is $\mathcal{H}^0 = v_0 \boldsymbol{\tau} \cdot \boldsymbol{p}$ with energy dispersion $E^0(\boldsymbol{p}) = \pm v_0 |\boldsymbol{p}|$, the constant energy contours of Eq. (2) are not circles but ellipses, as illustrated in Fig. 1.