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Physics and the choice of regulators in functional renormalisation group flows

Jan M. Pawlowski [a,b], Michael M. Scherer [a], Richard Schmidt [c,d], Sebastian J. Wetzel [a,*]

[a] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*
[b] *ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung mbH, Planckstr. 1, D-64291 Darmstadt, Germany*
[c] *ITAMP, Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA*
[d] *Physics Department, Harvard University, 17 Oxford Street, Cambridge, MA 02138, USA*

HIGHLIGHTS

- We show the regulator dependence of functional renormalisation (FRG) group equations.
- Solving FRG equations along closed loops provides a distance between two regulators.
- We suggest a practical procedure for devising optimal regulators.
- A universality class crossover can be induced by changing relative cutoff scales.
- We examine the regulator dependence explicitly in the Fermi polaron problem.

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ABSTRACT

The Renormalisation Group is a versatile tool for the study of many systems where scale-dependent behaviour is important. Its functional formulation can be cast into the form of an exact flow equation for the scale-dependent effective action in the presence of an infrared regularisation. The functional RG flow for the scale-dependent effective action depends explicitly on the choice of regulator, while the physics does not. In this work, we systematically investigate three key aspects of how the regulator choice affects RG flows: (i) We study flow trajectories along closed loops in the space of action functionals varying both, the regulator scale and shape function. Such a flow does not vanish in the presence of truncations. Based on a definition of the length of an RG trajectory, we suggest a constructive procedure for devising optimised

* Corresponding author.
E-mail address: wetzel_s@thphys.uni-heidelberg.de (S.J. Wetzel).

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regularisation schemes within a truncation. (ii) In systems with various field variables, a choice of relative cutoff scales is required. At the example of relativistic bosonic two-field models, we study the impact of this choice as well as its truncation dependence. We show that a crossover between different universality classes can be induced and conclude that the relative cutoff scale has to be chosen carefully for a reliable description of a physical system. (iii) Non-relativistic continuum models of coupled fermionic and bosonic fields exhibit also dependencies on relative cutoff scales and regulator shapes. At the example of the Fermi polaron problem in three spatial dimensions, we illustrate such dependencies and show how they can be interpreted in physical terms.

1. Introduction

In the past twenty years, the functional renormalisation group (FRG) approach [1] has been established as a versatile method allowing to describe many aspects of different physical systems in the framework of quantum field theory and statistical physics. Applications range from quantum dots and wires, statistical models, condensed matter systems in solid state physics and cold atoms over quantum chromodynamics to the standard model of particle physics and even quantum gravity. For reviews on the various aspects of the functional RG see [2–19].

The functional renormalisation group approach can be set-up in terms of an exact flow equation for the effective action of the given theory or model [1]. The choice of the initial condition at some large ultraviolet cutoff scale, typically a high momentum or energy scale, together with that of the regulator function determines both, the physics situation under investigation as well as the regularisation scheme. The functional RG flow for the scale-dependent effective action depends explicitly on the choice of regulator, while the physics does not. The latter is extracted from the full quantum effective action at vanishing cutoff scale implying a vanishing regulator. Hence, at this point no dependence on the choice of regulator is left, only the implicit choice of the regularisation scheme remains.

Typically, for the solution of the functional flow equation for the effective action one has to resort to approximations to the effective action as well as to the flow. Such a truncation of the full flow usually destroys the regulator independence of the full quantum effective action at vanishing cutoff. Therefore, devising suitable expansion schemes and regulators is essential for reliable results. In the context of perturbation theory and beyond, the principle of minimal sensitivity (PMS) [20] suggests to use extrema in the regulator dependence of specific observables as the most accurate result, see, e.g., [21–24]. Here, we strive for a constructive approach for finding an optimised regulator. Moreover, related considerations also allow for a discussion of the systematic error within a given truncation scheme. This has been examined in detail for the computation of critical exponents in models with a single scalar field in three dimensions within the lowest order of the local potential approximation (LPA), [7,25–33]: an optimisation procedure, [7,23,27] suggests a particular regulator choice – the flat regulator – which is also shown to yield the best results for the critical exponents.

The optimisation framework in [7] has been extended to general expansion schemes in a functional optimisation procedure including fully momentum-dependent approximation schemes. An application to momentum-dependent correlation functions in Yang–Mills theory and ultracold atoms can be found in [34–36].

Still, for more elaborate truncations, in particular higher orders of the derivative expansion in the LPA, including, e.g., momentum dependencies or higher-order derivative terms, little has been done when it comes to a practical implementation of constructive optimisation criteria. Also, more complex physical models with different symmetries such as, e.g., non-relativistic systems, or models with several different fields, for example mixed boson–fermion systems, demand for a thorough study of their regulator dependence in order to extract the best physical results from a given truncation.