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Enhancing multi-step quantum state tomography by PhaseLift

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HIGHLIGHTS

- PhaseLift is firstly applied in the adaptive quantum state tomography and the required store space of measurement operator is reduced.
- Traditional adaptive quantum state tomography is extended to three-step adaptive quantum state tomography.
- A new scheme of copy distribution of state is proposed to effectively use the limited copies of state.
- For several tested states, the mean square error of the reconstructed state is reduced to 10^{-9} from 10^{-4}.
- Fidelity is more accurately estimated with limited copies of state compared with traditional adaptive quantum state tomography.

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ABSTRACT

Multi-photon system has been studied by many groups, however the biggest challenge faced is the number of copies of an unknown state are limited and far from detecting quantum entanglement. The difficulty to prepare copies of the state is even more serious for the quantum state tomography. One possible way to solve this problem is to use adaptive quantum state tomography, which means to get a preliminary density matrix in the first step and revise it in the second step. In order to improve the performance of adaptive quantum state tomography, we develop a new distribution scheme of samples and extend it to three steps, that is to correct it once again based on the density matrix obtained in the traditional adaptive quantum state tomography. Our numerical results show that the mean square error of the reconstructed density matrix by our new method is improved to the level from 10^{-4} to

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10^{-9} for several tested states. In addition, PhaseLift is also applied to reduce the required storage space of measurement operator.

1. Introduction

Quantum entanglement is a necessary resource for quantum teleportation [1] and quantum computation [2]. It has been realized in photons, atoms and superconducting quantum circuits [2–4]. Certification of the entanglement of a state requires fidelity calculation. To further characterize the entanglement of a quantum state, an identical density matrix is an indispensable tool. Determination of this unique density matrix requires quantum measurements. Due to the characteristics of the destructiveness of the photon measurement, many copies of this unknown state are prepared to realize the quantum measurement. So far, it has been a challenge to prepare enough samples of the entangled state for detecting multi-photon entanglement in experiment [3]. Therefore a scheme is required which consumes copies of a state as few as possible and thus making the prepared copies of the state are sufficient to confirm entanglement or reconstruct a density matrix. The whole process to reconstruct the density matrix is named as quantum state tomography (QST) [5]. Specifically, the number of counts corresponding to different positive operator-valued measures (POVMs) is gained by repeatedly measuring the copies of a state. The ratio of the number of counts in one basis to the number of all counts detected in the same measurement setting is defined as a relative frequency, which can be obtained when the number of copies of the state is large enough. The relative frequencies are approximately regarded as probabilities. This measurement process follows Born's rule [6]. Then these probabilities can determine a density matrix by inverse transformation.

In QST, three main problems are encountered.

I, Several best measurement settings are required to be chosen for estimating the density matrix. Platonic solid measurements (the measurement bases are the vectors that connect the centers of the faces of the platonic solid and the center point in the Bloch sphere) give good performance of the reconstruction, and the overcomplete measurement sets can be commonly used to improve the accuracy of tomographic reconstruction [7]. However, these theoretical results are not easily accomplished in experiment, since the platonic solid measurements of overcomplete sets takes too much time in tomography.

II, Based on the measurement results (relative frequencies), it requires much time to reconstruct a multi-qubit density matrix. Hence, efficient algorithms are required to reconstruct a density matrix, the algorithms such as Maximum Likelihood (ML) estimation and the Least Squares (LS) method are commonly applied to QST [8]. Recently, Compressive Sensing (CS) is also utilized to conduct tomography [9]. Its performance is compared with ML in experiment [10], which shows that ML is better than CS. However, the conclusion may be taken with due care since the property of a density matrix is considered as the constraint of ML while it is not taken into account in the CS. Besides, a numerical result shows that CS outperforms LS when sampling rate is low [11], while the opposite is true when sampling rate is high [11]. Based on the special characteristics of POVMs, PhaseLift [12,13] is a good approach to save the storage space of POVMs, and has been also applied to solve a density matrix when the state is close to a pure state [14].

III, The number of copies of an unknown state measured in each setting may not be equal. Identical copies of the arbitrary unknown quantum state are evenly distributed in each measurement setting in general tomography experiments. However, the required copies of a state increase sharply with the dimensions of the Hilbert space for the quantum states. Therefore a scheme needs to be explored to cut down a required number of copies of the unknown state in QST. Recently, the adaptive QST is proposed to solve this problem [15,16]. It divides the measurement into two steps. The first step is to get a preliminary density matrix and the second step is to correct it by measurements of the diagonal basis of the density matrix obtained previously [15]. It reduces the infidelity between the estimated and the true state from $O(1/\sqrt{N})$ to $O(1/N)$, where N is the total number of samples of the unknown state [15]. This idea is further substantiated by one-qubit experiment [17]. The density matrix is reconstructed twice in the experiment.