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Universal properties of the FQH state from the topological entanglement entropy and disorder effects

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HIGHLIGHTS

- The TEE in Read–Rezayi FQH series are studied on cylinder in real space.
- Ground state TEE is robust before gap closing in the presence of the disorder.
- Strong disorder drives the system into a MBL phase.

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ABSTRACT

The topological entanglement entropy (TEE) is a robust measurement of the quantum many-body state with topological order. In fractional quantum Hall (FQH) state, it has a connection to the quantum dimension of the state itself and its quasihole excitations from the conformal field theory (CFT) description. We study the entanglement entropy (EE) in the Moore–Read (MR) and Read–Rezayi (RR) FQH states. The non-Abelian quasihole excitation induces an extra correction of the TEE which is related to its quantum dimension. With considering the effects of the disorder, the ground state TEE is stable before the spectral gap closing and the level statistics seems to have significant change with a stronger disorder, which indicates a many-body localization (MBL) transition.

1. Introduction

Fractional quantum Hall (FQH) liquids are remarkable many-electron systems that occur in two-dimensional electron gas with a perpendicular magnetic field [1]. This is the most studied and first

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experimentally realized system in condensed matter physics that has the topological order [2]. It is a typical strong correlated electron system with quenched kinetic energies by magnetic field which fails the application of the perturbation theory. On the other hand, comparing with the Landau theory of the quantum phase transition, there is no order parameter, or symmetry breaking to describe the phase transition between any two FQH states. Therefore, the understanding of the FQH effect has only benefited either from the numerical diagonalizing the Hamiltonian for finite size system, such as exact diagonalization [3], density matrix renormalization group [4–6], matrix product state [7,8], et al., or using of model wavefunctions [9–11]. In a seminal paper of Moore and Read [11], it was found that these model wavefunctions can be expressed as correlators of the electron operators in a conformal field theory (CFT), i.e., the so called conformal blocks. Although the model wavefunctions are not the exact ground state wavefunctions of a realistic Hamiltonian, they are supposed to capture the universal properties of the FQH states such as fractional quasiparticle excitations and their fusion relations, statistics, as well as exponents in the edge tunneling and quantum dimensions for quasiparticles. The most striking theoretically predicted properties of the FQH quasiparticles are the emergence of the Abelian or non-Abelian braiding statistics [11–13]. The interchange of two Abelian quasiparticles adds a nontrivial phase on the wavefunction. They are named "anyons" since the phase is neither π by fermions nor 2π by bosons. The typical Abelian FQH states are the Laughlin series at $\nu = 1/3, 1/5, 2/3 \cdots$. However, interchange two non-Abelian quasiparticles results in a ground state unitary transformation in the topological degenerate Hilbert space. According to this, the non-Abelian FQH states have received much interests due to their potential applications in the topological quantum computation [14–16] recently. Thus far there are two most interesting examples as the candidates for the non-Abelian states which have been realized in experiments, namely the FQH states on the first Landau level at $\nu = 5/2$ [17] and $\nu = 12/5$ [18,19]. For the even denominator FQH state at $\nu = 5/2$, Moore and Read [11] proposed a p-wave paired wavefunction as a candidate ground state. The nature of the 12/5 state is still undetermined. However, the most exciting candidate of the ground state is the $k = 3$ parafermion state proposed by Read and Rezayi [13] which describes a condensate of three-electron clusters.

For the FQH states, it has been established that there is a deep connection between the bipartite EE [20], or entanglement spectrum [21] and the topological properties embedded in the ground state and its low-lying excitations. This connection is based on the CFT description of the FQH model wavefunctions. For topological states in two dimensional systems, the bipartite EE satisfies the "area law" with a universal order $O(1)$ correction, namely the TEE [22–24], i.e., $S = \alpha L - \gamma_t$ where the L is the length of the boundary between two subsystems. For example, the bipartite FQH system can be implemented in both the momentum and real space for the two dimensional electron system. The former is called the orbital cut (OC) [20] and the latter real space cut (RC) [25]. In this work we mostly use the RC since it has a more accurate definition of the boundary in the "area law". The EE depends on the way of the partition of the system, or α is not universal. However, the TEE γ_t is a robust measurement of quantum entanglement in a topological phase. It has a connection to the total quantum dimension as $\gamma_t = \ln \mathcal{D}$ and $\mathcal{D} = \sqrt{\sum_i d_i^2}$, where d_i s are the quantum dimensions of each sector making up the topological field theory of the corresponding FQH states. In fact, for a general RR state with order-k clustering and at filling fraction $\nu = \frac{k}{kM+2}$, the total quantum dimension is $\mathcal{D}_{k,M} = \frac{\sqrt{(k+2)(kM+2)}}{2\sin[\frac{\pi}{k+2}]}$. Such as the Laughlin state at $\nu = 1/3$, the MR state at $\nu = 5/2$ and the RR state at $\nu = 12/5$ are corresponding to $M = 1$ and $k = 1, 2, 3$ RR states respectively. The TEE has an additional correction when a topological excitation, or a quasihole is created in the system, i.e., $\gamma_t^{qh} = \ln \mathcal{D} - \ln d_\alpha$ where d_α is the quantum dimension of the quasihole. In general, Abelian quasihole excitation has quantum dimension $d_\alpha = 1$ and $d_\alpha > 1$ for non-Abelian ones. Therefore, the behaviors of the EE, especially the TEE should be very different while a non-Abelian quasihole is created, in other words, we can measure the quantum dimension of the quasihole from the shift of the EE before and after its excitation.

The discussions above are based on a clean system without introducing the effects of the disorder. The topological properties are believed to be robust in the presence of a weak disorder. When the strength of the disorder is comparable to that of the interaction between electrons, the FQH will eventually be destroyed and the system enters into a localized insulating phase. The topological Chern