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Penetration of fast projectiles into resistant media: From macroscopic to subatomic projectiles

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HIGHLIGHTS

- Basic principles of the penetration of projectiles of any size or velocity.
- Newton's inertial theory of resistance can be extended to nano-projectiles.
- How the maximum velocity of a hypersonic projectile is determined by its strength.
- Transition from subatomic to macroscopic projectiles in terms of projectile velocity.

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ABSTRACT

The penetration of a fast projectile into a resistant medium is a complex process that is suitable for simple modeling, in which basic physical principles can be profitably employed. This study connects two different domains: the fast motion of macroscopic bodies in resistant media and the interaction of charged subatomic particles with matter at high energies, which furnish the two limit cases of the problem of penetrating projectiles of different sizes. These limit cases actually have overlapping applications; for example, in space physics and technology. The intermediate or mesoscopic domain finds application in atom cluster implantation technology. Here it is shown that the penetration of fast nano-projectiles is ruled by a slightly modified Newton's inertial quadratic force, namely, $F \sim v^{2-\beta}$, where β vanishes as the inverse of projectile diameter. Factors essential to penetration depth are ratio of projectile to medium density and projectile shape.

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1. Introduction

The analytical study of the resistance to the motion of projectiles begins with *Book Two* of Newton's *Principia*, entitled *The motion of bodies (in resisting mediums)* [1]. Other classics have studied this subject, which has obvious applications, for example, military applications. In this regard, we can mention the classic treaty on gunnery by Robins [2]. His work was continued by Euler [3]. Later, Poncelet [4] and Resal [5] deduced formulas for penetration depth that are still in use. The simplest formula for penetration depth is due to Gamow [6] but he attributed it to Newton [7,8]. Of course, the various aspects of the impact and penetration of projectiles in resistant media are treated in several modern books and reviews [9–15].

Newton's theory of resistance focuses on fluids [1], but it can be applied to the motion of *fast* projectiles in any media [6]. Projectiles with velocities about 1 km/s are fairly normal, while larger velocities, about 10 km/s, are typical in space and, in fact, constitute a hazard in space engineering [16–20]. The range of velocities of fast macroscopic projectiles to consider goes from a variable and medium-dependent lower limit, which will be determined, to a less variable upper limit, about 10 km/s, which is determined, in essence, by basic atomic physics.

The study of the penetration of fast subatomic particles in matter is relatively recent, of course. Bohr devised an essentially correct theory in 1913 and completed it along the following years [21]. The research on the many aspects of particle penetration has played an important role in the development of modern physics and the theory is now well established [22]. It is studied in various contexts, from fundamental particle physics to areas of applied physics, such as nuclear engineering or medicine, solid state physics, etc. In particular, high-energy subatomic particles are part of the space environment, as well as fast meteoroids [20].

Between subatomic and macroscopic projectiles, there is a *mesoscopic* range of *nano-projectiles*, with important technological applications [23]. The few studies of their relation to macroscopic projectiles [24,25] only treat particular aspects of the problem. Here we study the problem of resistance to projectile penetration within a unified conceptual framework that applies to the full ranges of sizes and velocities, and we deduce some novel and useful facts, especially applicable to nano-projectiles.

For the sake of simplicity, we disregard the effects of the impact of the projectile on the surface of the medium. If the penetration depth is considerable, the surface effects (spalling, sputtering, etc.) are not significant. Another complication is the possible deformation or fragmentation of the projectile, considered in Section 3.3. These effects reduce the penetration depth. As we will see, a fast projectile penetrates deeply when the density and strength of the projectile are considerably higher than those of the medium. This condition is less strict for streamlined projectiles. Subatomic particles are necessarily very fast (with velocities larger than 2000 km/s) and also are very penetrating (for their size).

We begin by setting the basic framework for projectile penetration in Section 2, including a dimensional analysis, which allows us to connect with Gamow's formula for penetration of a fast projectile. We need to define "fast" by introducing a critical velocity. Next, we analyze the physics of resistance (Section 3). The analysis of resistance to supersonic projectiles leads us to consider complex high temperature and high pressure phenomena. Next, in Section 4, the penetration of charged subatomic particles in matter is treated in analogy with that of macroscopic projectiles, beginning by a dimensional analysis. A deeper analysis of the electrodynamic origin of resistance follows (Section 5). Finally, we study nano-projectiles, as the nexus of the macroscopic and atomic theories of penetration (Section 6).

Our subject encompasses two fields of physics that are studied by different communities and requires concepts of both fields, some of which are well known by the corresponding community but probably not by the other. For this reason, it is preferable to refer to basic articles and textbooks, when possible. An apology is due to expert readers, who may find some concepts and references too basic.

A note on notation. We employ often two signs for the asymptotic equivalence of functions: $f \sim g$ means that the limit of $f(x)/g(x)$ is finite and non-vanishing when x tends to some value (or to infinity), whereas $f \approx g$ means, in addition, that the limit is one. We also use the signs $\propto$ and $\simeq$, which