ARTICLE IN PRESS

Annals of Physics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Scaling behaviour and superconducting instability in anisotropic non-Fermi liquids

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ARTICLE INFO

Article history:
Received 28 September 2016
Accepted 21 November 2016
Available online xxxx

Keywords:
Non-Fermi-liquid SDW anisotropic scaling

ABSTRACT

We study the scaling behaviour of the optical conductivity (σ), free energy density (F) and shear viscosity of the quantum critical point associated with spin density wave phase transition for a two-dimensional metallic system with C_2 symmetry. A non-Fermi liquid behaviour emerges at two pairs of isolated points on the Fermi surface due to the coupling of a bosonic order parameter to fermionic excitations at those so-called "hot-spots". We find that near the hot-spots, σ and F obey the scalings expected for such an anisotropic system, and the direction-dependent viscosity to entropy density ratio is not a universal number due to the anisotropy. Lastly, we also estimate the effect of the fermion–boson coupling at the hot-spots on superconducting instabilities.

1. Introduction

The "strange metal" phase observed in numerous correlated electron compounds, for example the cuprates, are unconventional metallic states that cannot be studied using the framework of the Landau–Fermi liquid theory, as the quasiparticle excitations get destroyed due to their coupling with some gapless boson. There have been intensive efforts to study such "non-Fermi liquid" states [1–30]. These states may involve the gapless bosons carrying either (1) zero momentum, such as the Ising-nematic critical point [7,9,10,12,20,24,26–29,31–45] and nonrelativistic fermions coupled with an emergent gauge field [46–51]; or (2) non-zero momenta, such as the spin density wave (SDW) and charge density wave (CDW) critical points [13–15,21–23,30].

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http://dx.doi.org/10.1016/j.aop.2016.11.009
0003-4916/

Recently, a “co-dimensional regularization scheme” has been developed for a perturbatively controlled study of the SDW critical point in two-dimensional metals with four-fold (C_4) [21] and (C_2) [23] symmetries, by embedding the one-dimensional Fermi surface in a higher dimensional space. These are non-Fermi liquid systems where the critical theory is described by isolated points called “hot-spots”, such that a bosonic order parameter is coupled to fermionic excitations at four (two) pairs of hot-spots around the Fermi surface with C_4 (C_2) symmetry. In the second case, the C_4-symmetric metallic state is explicitly or spontaneously broken to a C_2-symmetric one [52–60], and an anisotropic non-Fermi liquid emerges when the system undergoes a continuous density wave transition [61–65].

The hot-spot contribution to optical conductivity and finite temperature free energy density for the C_4-symmetric SDW critical point has been found in Ref. [22] using the regularization scheme of Ref. [21], where the authors concluded that hyperscaling is obeyed near the hot-spots.[1] This is expected for non-Fermi liquids arising from the interaction of the Fermi surface with bosons carrying non-zero momentum, where all the hot-spots exhibit an isotropy. In the present work, we compute the optical conductivity (σ) and free energy density for the anisotropic C_2-symmetric case, using the ϵ-expansion of Ref. [23]. Furthermore, we calculate the shear viscosity (η) and find the scaling behaviour of the ratio between η and entropy density (s). One can carry out a Boltzmann analysis directly in $d = 2$, which will be very similar to the C_4-symmetric SDW case considered in Ref. [22]. From such computations, it can be shown that the leading order temperature (T) dependence of the quantum critical conductivity (σ_Q) has the same form as the frequency (ω) dependence of σ. The T-dependence of the DC viscosity can also be inferred from the frequency dependent shear viscosity computed from the field theory. Lastly, we also estimate the effect of the fermion–boson coupling at the hot-spots on superconducting instabilities.

The paper is organized as follows: In Section 2, we review the co-dimensional regularization procedure devised in Ref. [23] to obtain a perturbative control of the C_2-symmetric SDW quantum critical point. In Section 3, we compute the scaling of the optical conductivity with frequency. Section 4 deals with the calculation of the finite temperature free energy density. In Section 6, we address the question whether the fermion–boson coupling results in an enhancement of the instability of four-fermion interactions to superconducting pairing. The expressions for the direction-dependent viscosity to entropy density ratios have been derived in Section 5. We conclude with a summary and outlook in Section 7. The detailed computation of the current–current correlators has been shown in the Appendix.

2. Model

The action describing the fermions confined to two spatial dimensions and interacting with an SDW order parameter is given by [23]:

$$
\begin{aligned}
S = & \sum_{j=1}^{N_f} \sum_{\sigma=1}^{N_c} \sum_{l=1}^{2} \sum_{m=\pm} \int \frac{d^3k}{(2\pi)^3}\, \psi^*_{l,m,j,s}(k)\left(ik_0 + \mathcal{E}_{l,m}(k)\right)\psi_{l,m,j,s}(k) \\
& + \frac{1}{4}\int \frac{d^3q}{(2\pi)^3}\left(q_0^2 + q_x^2 + c^2 q_y^2\right)\mathrm{Tr}\left(\Phi(-q)\Phi(q)\right) \\
& + \frac{g}{\sqrt{N_f}} \sum_{j=1}^{N_f} \sum_{l=1}^{2} \sum_{\sigma,\sigma'=1}^{N_c} \int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\left[\psi^*_{l,+,j,\sigma}(k+q)\,\Phi_{\sigma,\sigma'}(q)\,\psi_{l,-,j,s'}(k) + \text{h.c.}\right] \\
& + \frac{1}{4}\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\Big[u_{1;0}\,\mathrm{Tr}\left(\Phi(-q_1+q_2)\Phi(q_1)\right)\,\mathrm{Tr}\left(\Phi(-q_3-q_2)\Phi(q_3)\right) \\
& + u_{2;0}\,\mathrm{Tr}\left(\Phi(-q_1+q_2)\Phi(q_1)\Phi(-q_3-q_2)\Phi(q_3)\right)\Big],
\end{aligned}
\tag{2.1}
$$

[1] In a recent work [30], the authors have employed a non-perturbative treatment of the problem and found that there is hyperscaling violation for the free energy density in two spatial dimensions.

Please cite this article in press as: I. Mandal, Scaling behaviour and superconducting instability in anisotropic non-Fermi liquids, Annals of Physics (2016), http://dx.doi.org/10.1016/j.aop.2016.11.009