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Exact solution of the two-axis countertwisting Hamiltonian

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ABSTRACT

It is shown that the two-axis countertwisting Hamiltonian is exactly solvable when the quantum number of the total angular momentum of the system is an integer after the Jordan–Schwinger (differential) boson realization of the SU(2) algebra. Algebraic Bethe ansatz is used to get the exact solution with the help of the SU(1,1) algebraic structure, from which a set of Bethe ansatz equations of the problem is derived. It is shown that solutions of the Bethe ansatz equations can be obtained as zeros of the Heine–Stieltjes polynomials. The total number of the four sets of the zeros equals exactly $2J + 1$ for a given integer angular momentum quantum number J, which proves the completeness of the solutions. It is also shown that double degeneracy in level energies may also occur in the $J \to \infty$ limit for integer J case except a unique non-degenerate level with zero excitation energy.

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1. Introduction

Squeezed spin states of both Bose and Fermi many-body systems [1–8], where a component of the total angular momentum of an ensemble of spins has less uncertainty [9,10] than other cases

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without quantum mechanical correlations, have been attracting great attention [11–14], not only because they are intrinsically interesting, but also because of being practically useful in precision measurements [2], quantum information, and fundamental tests of quantum mechanics [15]. As shown in [1], maximal squeezed spin states of a many-particle system can be generated by using the two-axis countertwisting mechanism, of which the Hamiltonian of the system is referred to as the two-axis countertwisting Hamiltonian. When the number of particles is small, the Hamiltonian can easily be diagonalized for a given quantum number of the total angular momentum of the system. However, one needs to handle a huge sparse matrix when the system contains an ensemble of a large number of particles [5–8,14]. Exact analytical solution to the problem should be helpful, especially when one deals with a large number of particles. As noted in [16], up till now, there has been no general analytic solution available, though there were a few analytic treatments [17–20] for a system with small number of particles.

In fact, significant progresses have been made in finding exact solutions of many-spin systems since the work of Bethe, Gaudin and Richardson [21–24]. Particularly, the Lipkin–Meshkov–Glick (LMG) model, which can be expressed in terms of the total angular momentum operators of the system up to their quadratic form, has been analytically solved by using the algebraic Bethe ansatz [25,26]. The same problem can also be solved by using the Dyson boson realization of the SU(2) algebra [27–29], of which the solutions may be obtained from the Riccati differential equations [27,28]. Discrete phase analysis of the model with applications to spin squeezing and entanglement was studied in [30]. In [31], it was shown that asymmetric rotor Hamiltonian can also be solved analytically by using the algebraic Bethe ansatz. However, though the two-axis countertwisting Hamiltonian is equivalent to a special case of the LMG model [27,28] after an Euler rotation, the procedures used in [25,26,31] cannot be applied to the two-axis countertwisting Hamiltonian directly.

In this work, we show that the two-axis countertwisting Hamiltonian is indeed exactly solvable when the quantum number of the total angular momentum of the system is an integer after the Jordan–Schwinger (differential) boson realization of the SU(2) algebra. Similar to [31], exact solution to the problem will be derived based on the SU(1,1) algebraic structure after suitable transformations. Moreover, it is shown that solutions of the Bethe ansatz equations can be obtained from zeros of the Heine–Stieltjes polynomials, which, in turn, verifies the completeness of the solutions.

2. The two-axis countertwisting Hamiltonian

The two-axis countertwisting Hamiltonian may be written as [1]

$$H_{\mathrm{TA}} = \frac{\chi}{2i}(J_+^2 - J_-^2), \tag{1}$$

where $J_\pm$ are the angular momentum raising and lowering operators, $i = \sqrt{-1}$, and χ is a constant. The Hamiltonian (1) is invariant under both parity and time reversal transformations, namely, it is PT-symmetric. Due to time-reversal symmetry, similar to the asymmetric rotor case [31], level energies of the system are all doubly degenerate when the quantum number of the total angular momentum is a half-integer. (1) is also equivalent to a special LMG Hamiltonian after rotation of the system by $\pi/4$ around z axis, of which the thermodynamic limit was studied in [27,28] by using the Dyson boson (differential) realization and the corresponding Riccati differential equations.

Using the Jordan–Schwinger realization of SU(2), we have

$$J_+ = a^\dagger b, \qquad J_- = b^\dagger a, \qquad J_0 = \frac{1}{2}(a^\dagger a - b^\dagger b), \tag{2}$$

where a, b and $a^\dagger, b^\dagger$ are boson annihilation and creation operators introduced. It can be observed that eigenstates of (1) may be expressed as

$$|N, \zeta\rangle = F_t^{(\zeta)}(a^{\dagger 2}, b^{\dagger 2})|\nu_a, \nu_b\rangle \tag{3}$$

after the Jordan–Schwinger realization, where $F_t^{(\zeta)}(a^{\dagger 2}, b^{\dagger 2})$ is a homogeneous polynomial of degree t with variables $\{a^{\dagger 2}, b^{\dagger 2}\}$, $|\nu_a, \nu_b\rangle$ is the boson pairing vacuum satisfying $a^2|\nu_a, \nu_b\rangle = 0$, $b^2|\nu_a, \nu_b\rangle = 0$,

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