ARTICLE IN PRESS

Annals of Physics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Comment on: “Ground state energies from converging and diverging power series expansions”, Ann. Phys. 373 (2016) 456–469

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ARTICLE INFO

Article history:
Received 9 September 2016
Accepted 6 December 2016
Available online xxxx

Keywords:
Finite attractive well
Perturbation expansion
Asymptotic expansion
Periodic boundary conditions
Square potential
Dirac delta potential

ABSTRACT

We compare two alternative expansions for finite attractive wells. One of them is known from long ago and is given in terms of powers of the strength parameter. The other one is based on the solution of the equations of the Rayleigh–Schrödinger perturbation theory in a basis set of functions of period L. The analysis of exactly solvable models shows that although the exact solution of the problem with periodic boundary conditions yields the correct result when $L \to \infty$ the coefficients of the series for this same problem blow up and fail to produce the correct asymptotic expansion.

1. Introduction

In a recent paper Lisowski et al. [1] proposed the application of an approximate method for the treatment of the Schrödinger equation with finite attractive potentials. It consists of solving the secular equation for the matrix representation of the Hamiltonian operator in a basis set of functions of period L. The eigenvalues of this matrix are expected to approach the actual eigenvalues of the problem in the limit $L \to \infty$. The authors also applied the same approach to the equations given by perturbation theory thus obtaining approximate perturbation coefficients that depend on the box

DOI of original article: http://dx.doi.org/10.1016/j.aop.2016.07.012.

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http://dx.doi.org/10.1016/j.aop.2016.12.016
0003-4916/

Please cite this article in press as: P. Amore, F.M. Fernández, Comment on: “Ground state energies from converging and diverging power series expansions”, Ann. Phys. 373 (2016) 456–469, Annals of Physics (2016), http://dx.doi.org/10.1016/j.aop.2016.12.016

length L. They argued that this perturbation series is convergent when the width of the ground-state eigenfunction is larger than L and divergent when that width is smaller than L. The former case takes place when the potential strength $V_0 > 0$ is smaller than a critical value V_{crit} and the latter when $V_0 > V_{crit}$. In order to sum the perturbation series in both regimes the authors proposed the application of the well known Borel transformation with the substitution of a finite integral for the infinite one when carrying out the inverse transformation.

It is well known that there exists a perturbation series about $V_0 = 0$ in the case of a one-dimensional short-range potential and there are even explicit expressions for the first perturbation coefficients [2] (and references therein). However, Lisowski et al. [1] state that "It is often assumed that bound states of quantum mechanical systems are intrinsically non-perturbative in nature and therefore any power series expansion methods should be inapplicable to predict the energies for attractive potentials". The purpose of this paper is to discuss the connection between the approximate perturbation series proposed by Lisowski et al. [1] and the well known exact perturbation series [2].

In Section 2 we outline the application of the two perturbation methods just mentioned to a general finite attractive well. In Section 3 we discuss the perturbation series by means of simple, exactly solvable models. Finally, in Section 4 we summarize the main results and draw conclusions.

2. Short-range shallow wells

Throughout this paper we consider a particle of mass m that moves in one dimension under the effect of a short-range negative potential $-V_0 \leq V(x) \leq 0$ that we suppose to be of even parity $V(-x) = V(x)$. In order to simplify the calculations it is convenient to rewrite the Hamiltonian operator

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x), \tag{1}$$

in dimensionless form by means of the change of variables $x = \gamma x'$, where γ is a suitable length. In this way we obtain

$$H' = \frac{2m\gamma^2}{\hbar^2}H = -\frac{d^2}{dx'^2} + \lambda v(x'),$$
$$\lambda = \frac{2m\gamma^2 V_0}{\hbar^2},\quad v(x') = \frac{V(\gamma x')}{V_0}. \tag{2}$$

Thus an eigenvalue E of H and the corresponding one ϵ of H' are related by

$$\epsilon = \frac{2m\gamma^2}{\hbar^2}E. \tag{3}$$

From now on we omit the prime on the dimensionless quantities and write the Hamiltonian operator as

$$H = -\frac{d^2}{dx^2} + \lambda v(x), \tag{4}$$

where $-1 \leq v(x) \leq 0$.

It is well known that in the case of a short-range potential the ground state energy can be expanded in a formal perturbation series of the form [2]:

$$-(-\epsilon)^{1/2} = \frac{\lambda}{2}\int v(x)\,dx + \frac{\lambda^2}{4}\int\int v(x)v(y)|x-y|\,dxdy + \frac{\lambda^3}{48}\int\int\int v(x)v(y)v(z)\,(|x-y|+|y-z|+|z-x|)^2\,dxdydz + \frac{\lambda^4}{96}\int\int\int\int v(x)v(y)v(z)v(t)\,\big(|x-y|^3$$

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