Annals of Physics 382 (2017) 85–111

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Quantum phases and entanglement properties of an extended Dicke model

Michal Kloc *, Pavel Stránský, Pavel Cejnar

Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, Prague, 18000, Czech Republic

HIGHLIGHTS

- Dicke-like superradiance model with controllable breaking of integrability is studied.
- Thermal and excited-state quantum phase transitions (ESQPTs) are identified.
- Quantum phases are characterized by energy variations of smoothed expectation values.
- Atom–field and atom–atom entanglement is studied across wide excitation spectrum.
- Anomalies of entanglement are observed at certain ESQPT borderlines.

ARTICLE INFO

Article history:
Received 7 September 2016
Accepted 15 April 2017
Available online 29 April 2017

Keywords:
Single-mode superradiance model
Excited-state quantum phase transitions
Thermal phases
Quantum phases
Entanglement properties of excited states

ABSTRACT

We study a simple model describing superradiance in a system of two-level atoms interacting with a single-mode bosonic field. The model permits a continuous crossover between integrable and partially chaotic regimes and shows a complex thermodynamic and quantum phase structure. Several types of excited-state quantum phase transitions separate quantum phases that are characterized by specific energy dependences of various observables and by different atom–field and atom–atom entanglement properties. We observe an approximate revival of some states from the weak atom–field coupling limit in the strong coupling regime.

1. Introduction

Since its prediction in 1954 [1], the effect of superradiance has attracted a lot of theoretical and experimental attention [2–4]. Its basic principle – the fact that a coherent interaction of an unexcited

* Corresponding author.
E-mail address: kloc@ipnp.troja.mff.cuni.cz (M. Kloc).

http://dx.doi.org/10.1016/j.aop.2017.04.005

gas with the vacuum of a common field can create a spontaneous macroscopic excitation of both matter and field subsystems – appears in various incarnations in diverse branches of physics [5,6].

The Dicke model [1,7,8] of superradiance resorts to a maximal simplification of the problem to capture the main features of the superradiant transition in the most transparent way. The model shows a thermal phase transition, analyzed and discussed in Refs. [9–12], as well as a zero-temperature (ground-state) Quantum Phase Transition (QPT) [13–15], which was addressed experimentally and realized with the aid of a superfluid gas in a cavity [16–18]. Recent theoretical analyses showed that the model exhibits also a novel type of criticality—the one observed in the spectrum of excited states [19–22]. These so-called Excited-State Quantum Phase Transitions (ESQPTs) affect both the density of quantum levels as a function of energy and their flow with varying control parameters, and are present in a wide variety of quantum models with low numbers of degrees of freedom [23–28].

In this work, we analyze properties of a simple superradiance model interpolating between the familiar Dicke [1] and Tavis–Cummings [7,8] Hamiltonians. A smooth crossover between both limiting cases is achieved by a continuous variation of a model parameter, which allows one to observe the system's metamorphosis on the way from the fully integrable (hence at least partly understandable) to a partially chaotic (so entirely numerical) regime. One of the aims of our work is to survey the phase-transitional properties of the extended model and to investigate the nature of its *quantum phases*—the domains of the excited spectrum in between the ESQPT critical borderlines. A usual approach to characterize different phases related to the ground state of a quantum system makes use of suitable "order parameters", i.e., expectation values of some observables. We show that an unmistakable characterization of phases involving excited states is not achieved through the expectation values alone but rather through their different smoothed energy dependences (trends).

The second part of our analysis is devoted to the *entanglement properties of excited states* across the whole spectrum and their potential links to the ESQPTs and quantum phases of the model. It is known that a continuous ground-state QPT in many models (including the present one) is characterized by a singular growth of entanglement within the system, which can be seen as a quantum counterpart of the diverging correlation length in continuous thermal phase transitions [29–36]. A question therefore appears whether there exist any entanglement-related signatures of ESQPTs. The extended Dicke model is rather suitable for a case study of this type since it allows one to analyze at once various types of entanglement—that between the field and all atoms, and that between individual atoms.

The plan of the paper is as follows: Basic quantum and classical features of the model are described in Section 2. Thermal and quantum critical properties and a classification of thermodynamic and quantum phases are presented in Section 3. The atom–field and atom–atom entanglement properties are investigated in Section 4. Conclusions come in Section 5.

2. Extended Dicke model

2.1. Hamiltonian, eigensolutions, classical limit

Consider single-mode electromagnetic field with photon energy ω (polarization neglected) interacting with an ensemble of N two-level atoms, all with the same level energies $\pm\omega_0/2$. The size of the atomic ensemble is assumed to be much smaller than the wavelength of photons (cavity size) so that all atoms interact with the field with the same phase. If we introduce an overall interaction strength λ and an additional interaction parameter δ (whose role will be explained later), the Hamiltonian can be written as

$$\begin{aligned} H &= \omega b^\dagger b + \omega_0 \sum_{k=1}^{N} \tfrac{1}{2}\sigma_z^k + \frac{\lambda}{\sqrt{N}}\left[\left(b^\dagger + b\right)\sum_{k=1}^{N}\tfrac{1}{2}\left(\sigma_+^k + \sigma_-^k\right) - (1-\delta)\sum_{k=1}^{N}\tfrac{1}{2}\left(b^\dagger\sigma_+^k + b\sigma_-^k\right)\right] \\ &= \underbrace{\omega\, b^\dagger b + \omega_0 J_z}_{H_{\rm free}} + \frac{\lambda}{\sqrt{N}}\underbrace{\left[b^\dagger J_- + bJ_+ + \delta\, b^\dagger J_+ + \delta\, bJ_-\right]}_{H_{\rm int}}, \end{aligned} \tag{1}$$

where operators $b^\dagger$ and b create and annihilate photons, while $\sigma_\bullet^k$ stands for the respective Pauli matrix with subscript $\{+, -, z\}$ or $\{x, y, z\}$ acting in the 2-state Hilbert space of the kth atom. The part of H