Annals of Physics 382 (2017) 127–135

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Type N Einstein space Time Machine spacetime

Faizuddin Ahmed

Hindustani Kendriya Vidyalaya, Dinesh Ojah Road, Guwahati-781005, India

HIGHLIGHTS

- A vacuum solution of the Einstein field equations with a Cosmological constant is presented.
- The spacetime admits closed timelike curves which appear after a certain instant of time in a causally well-behaved manner.
- The spacetime is a 4D generalization of flat Misner space in curved spacetime and the spacetime is axially symmetric.
- The spacetime admits a non-expanding, non-twisting, and shear-free geodesic null congruence and is of type N in the Petrov classification scheme.

ARTICLE INFO

Article history:
Received 24 September 2016
Accepted 9 April 2017
Available online 15 May 2017

Keywords:
Vacuum spacetime
Closed timelike curves
Misner space
Cosmological constant

ABSTRACT

We present an Einstein space axially symmetry spacetime admitting closed timelike curves (CTCs) which appear after a certain instant of time, *i.e.*, a time machine spacetime. The spacetime is a four-dimensional generalization of flat Misner space in curved spacetime, free-from curvature divergences and belongs to type N in the Petrov classification. The spacetime admits a non-expanding, non-twisting, and shear-free null geodesic congruence.

1. Introduction

The Einstein field equations of the Theory of General Relativity are a set of non-linear partial differential equations. It is very hard to find the exact solutions of the field equations without simplifying assumptions. Some known solutions of the field equations admit closed causal curves (CCCs) in the form of closed timelike curves (CTCs), closed timelike geodesics (CTGs) and closed null geodesics (CNGs). The presence of such curves in a spacetime violates the causality condition in General Relativity. The first solution of the field equations with causality violating curves, namely,

E-mail address: faizuddinahmed15@gmail.com.

http://dx.doi.org/10.1016/j.aop.2017.04.012

CTCs was the Gödel rotating-dust Cosmological model [1]. Another spacetime, that of van Stockum [2], which pre-dates the one given by Gödel, was shown later to have CTCs [3]. Other examples including wormhole models by Morris and collaborators [4,5], Gott's solution [6] of two infinitely-long cosmic string, Bonnor's rotating dust spacetime [7] and a pure radiation metric [8] have CTCs. Some well known vacuum spacetimes admitting CTC are the NUT-Taub metric [9] (see also [10]), Kerr and Kerr–Newmann black holes solution [11,12] (see also [13]), spacetime found by Ori [14,15], Bonnor metric [16,17], a maximally symmetric locally AdS metric [18] and an axially symmetric vacuum metric with naked singularity [19]. Some other spacetimes with CTC would be in [20–39].

There are other causality violating spacetimes that, in addition to CTCs, admit closed timelike geodesics and closed null geodesics. There have only been a handful of solutions with closed timelike geodesics found previously. These were discussed in detail in [39–45]. Another recent example shows the existence of closed null geodesics in a Ricci flat spacetime [46]. One way of classifying such causality violating spacetimes would be to categorize as either eternal time machine spacetime in which CTCs always exist (*e.g.* [1,2,11] etc.) or as time machine spacetime in which CTCs appear after a certain instant of time. In the latter category, the work of Ori and his collaborator [14,15,31,32] deserves special mention. However, many of the above models suffer from some severe physical problems for time machine spacetimes. For instance, in some of them, the Weak energy condition (WEC) is violated indicating unrealistic matter-energy source (*e.g.* [4,5,20,33,34,47]) and/or there is a curvature singularity [3,15,35,48]. The WEC states that for any physical (timelike) observer the energy density is non-negative, which is the case for all known types of (classical) matter fields. The time machine models discussed in [31,32,37,49] violate the Strong energy condition (SEC).

An eternal time machine spacetime is the one where CTCs form everywhere for some values of the radial coordinate and the spacetime does not admit a partial Cauchy surface and/or violate the Weak energy condition. The example of this category is Gödel's Cosmological model [1] where CTCs form everywhere for the radial coordinate $r > r_0 = \ln(1+\sqrt{2})$ and do not admit a partial Cauchy surface (an initial spacelike hypersurface). Tipler's rotating solution [3] and Mallett's solution [35] do not admit a partial Cauchy surface, and wormhole models [4,5] including some other models mentioned above violate the Weak energy condition. In time machine spacetimes, CTCs appear after a certain instant of time and are confined within some region called non-chronal region. There exists another region called chronal region where there are no closed causal (timelike or null) curves. The chronal region without CTCs of the spacetime is separated from a non-chronal (or bounded) region with CTCs by a Chronology horizon. Additionally, the time machine spacetimes should satisfy the basic requirements, namely, that the spacetime (i) admits an initial spacelike hypersurface, (ii) content known types of matter fields, (iii) obeys the Weak energy condition (also the other energy conditions) [13], (iv) is a four dimensional curved spacetime (not flat-space), and (v) is free-from curvature divergence. The time machine spacetime presented in [37] satisfies all the basic requirements, except the condition (ii), *i.e.*, the model does not fit any known types of matter fields and hence is not acceptable. A spacetime satisfies the above mention condition, and in addition, CTCs appear after a certain instant of time, would be physically acceptable as time machine spacetime than an eternal one. The possibilities that a naked curvature singularity gives rise to a Cosmic Time Machine have been discussed by Clarke and Felice [50] (see also [51–53]). A Cosmic Time Machine is a spacetime which is asymptotically flat and admits closed non-spacelike curves which extend to future infinity. Recently, the author and collaborators [19,54] constructed time machine spacetimes which may represent such Cosmic Time Machines.

In General Relativity, Chronological violating set in a spacetime $(\boldsymbol{M}, \boldsymbol{g})$ is a set of points through which closed timelike curves pass. The boundary of Chronology violating set is called the Cauchy horizon. Thus Cauchy horizon is a light-like boundary of the domain of dependence for which every causal curve passing through intersects the hypersurface exactly once. It can be caused either by the closed timelike curves, *i.e.*, the closed causal (timelike or null) curves might intersect the hypersurface more than once or by singularities. On one side of the Cauchy horizon, events in a spacetime cannot be causally connected. On the other side, events can be causally connected but the causal connection is Chronology violating (an event can come before its cause). A Chronology horizon is a special type of Cauchy horizon which separates spacetime a chronal region without CTCs to a non-chronal region with CTCs. A detailed discussion of the Cauchy horizon would be in [4,13,20,55–60].