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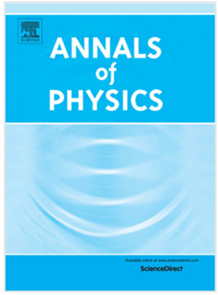

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Gaussian ensemble for quantum integrable dynamics

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We propose a Gaussian ensemble as a description of the long-time dynamics of isolated quantum integrable systems. Our approach extends the Generalized Gibbs Ensemble (GGE) by incorporating fluctuations of integrals of motion. It is asymptotically exact in the classical limit irrespective of the system size and, under appropriate conditions, in the thermodynamic limit irrespective of the value of Planck's constant. Moreover, it captures quantum corrections near the classical limit, finite size corrections near the thermodynamic limit, and is valid in the presence of non-local interactions. The Gaussian ensemble bridges the gap between classical integrable systems, where a generalized microcanonical ensemble is exact even for few degrees of freedom, and GGE, which requires thermodynamic limit. We illustrate our results with examples of increasing complexity.

The far from equilibrium dynamics of isolated many-body systems with many nontrivial integrals of motion attracted considerable attention as such dynamics have been recently realized in several experiments[1–7]. In particular, it has been conjectured that the infinite time averages of various observables for a system evolving with a time-independent Hamiltonian $\hat{H}$ are described by the Generalized Gibbs Ensemble (GGE)[8]:

$$\hat{\rho}_{\rm GGE} = Ce^{-\sum_i \beta_i \hat{H}_i}, \quad h_i \equiv \langle \hat{H}_i \rangle_0 = {\rm tr}(\hat{\rho}_{\rm GGE}\hat{H}_i), \quad (1)$$

where $\hat{H}_i$ is a complete (in some yet unspecified sense) set of integrals of motion for $\hat{H}$, the second equation relates β_i to expectation values h_i of the integrals in the initial state, and C is a normalization constant. A key difficulty with quantum GGE stems from the absence of an accepted well-defined notion of quantum integrability. As a result, GGE is strictly speaking unfalsifiable. For example, it was initially shown to fail for the 1D XXZ spin chain[9–11], but later studies[12,13] cured this by adding new integrals of motion in Eq. (1).

In contrast, classical integrability is well-defined[14]. Moreover, the microcanonical version of GGE – Generalized Microcanonical Ensemble (GME) is exact for a general classical integrable Hamiltonian $H(\boldsymbol{p},\boldsymbol{q})$[14,15],

$$\rho(\boldsymbol{p},\boldsymbol{q}) = C\prod_{k=1}^{n}\delta(H_k(\boldsymbol{p},\boldsymbol{q})-h_k), \quad (2)$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T O(t)dt = \int d\boldsymbol{p}d\boldsymbol{q}\, O(\boldsymbol{p},\boldsymbol{q})\rho(\boldsymbol{p},\boldsymbol{q}), \quad (3)$$

where $\boldsymbol{q}=(q_1,\dots,q_n)$ and $\boldsymbol{p}=(p_1,\dots,p_n)$ are the generalized coordinates and momenta and $H_k(\boldsymbol{p},\boldsymbol{q})$ are the integrals of motion. The time evolution of any dynamical variable (observable) $O(t)\equiv O(\boldsymbol{p}(t),\boldsymbol{q}(t))$ is obtained by evolving with $H(\boldsymbol{p},\boldsymbol{q})$ starting at $t=0$ and h_k is the initial value of $H_k(\boldsymbol{p},\boldsymbol{q})$. Note that unlike the microcanonical distribution for a nonintegrable Hamiltonian or GGE, Eq. (3) holds for any number of degrees of freedom n and arbitrary interactions, i.e. does not require thermodynamic limit. In some sense, classical integrable dynamics are more ergodic, but in a restricted part of the phase-space cut out by the integrals of motion.

What can play the role or replace the microcanonical ensemble for a quantum integrable system with arbitrary particle number? More precisely, how to quantize Eqs. (2) and (3), i.e. what is a suitable density matrix $\hat{\rho}$ that turns into Eq. (2) in $\hbar\to 0$ limit? To what extent does it describe the quantum dynamics and how does it compare to GGE? These are the questions we address in this paper. We argue that a minimal such $\hat{\rho}$ is a multivariable Gaussian in $\hat{H}_i$ and show that it has several remarkable features. This ensemble provides leading quantum corrections to the classical Eq. (2) for any number of degrees of freedom. It further yields leading finite size correction to GGE and is expected to work well in systems with long-range interactions, see also Fig. 1. We note that Gaussian ensembles were analyzed in literature in the context of both integrable[16,17] and non-integrable systems[18] as corrections to the corresponding Gibbs ensembles. In this work we show that the Gaussian ensemble applies to a generic class of systems, where GGE can completely fail. Although our focus is on integrable systems, we expect the Gaussian ensemble to apply equally well to chaotic systems with a few or none nontrivial integrals of motion besides the total energy.

In the case of Gibbs or Generalized Gibbs distributions, one can simply replace $H(\boldsymbol{p},\boldsymbol{q})$ or $H_k(\boldsymbol{p},\boldsymbol{q})$ with the corresponding operators. This does not work for Eq. (2), because the average of a product of $\hat{H}_i$ with so constructed density matrix is equal to the product of averages, which is not the case for a typical quantum state. Therefore, to reproduce various time averages, we need to broaden the delta-functions in Eq. (2).

It is natural to proceed by analogy with the usual quantum microcanonical ensemble and to replace the right hand side of Eq. (3) with an equal weight average over all eigenstates $|n\rangle$ of $\hat{H}_i$, $\hat{H}_i|n\rangle = E_i^{(n)}|n\rangle$, that have eigenvalues $E_i^{(n)}$ sufficiently close to quantum expectation values $h_i = \langle\hat{H}_i\rangle_0$ of the integrals in the initial state[19,20]. The problem is that $E_i^{(n)}$ are generally discrete, while h_i can be anywhere in between. For example, integrals