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Average subentropy, coherence and entanglement of random mixed quantum states

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ABSTRACT

Compact expressions for the average subentropy and coherence are obtained for random mixed states that are generated via various probability measures. Surprisingly, our results show that the average subentropy of random mixed states approaches the maximum value of the subentropy which is attained for the maximally mixed state as we increase the dimension. In the special case of the random mixed states sampled from the induced measure via partial tracing of random bipartite pure states, we establish the typicality of the relative entropy of coherence for random mixed states invoking the concentration of measure phenomenon. Our results also indicate that mixed quantum states are less useful compared to pure quantum states in higher dimension when we extract quantum coherence as a resource. This is because of the fact that average coherence of random mixed states is bounded uniformly, however, the average coherence of random pure states increases with the increasing dimension. As an important application, we establish the typicality of relative entropy of entanglement and distillable entanglement for a specific class of random bipartite mixed states. In particular, most of the random states in this specific class have relative entropy of entanglement and distillable entanglement equal to some fixed number (to within an arbitrary small error), thereby hugely reducing the complexity of computation of these entanglement measures for this specific class of mixed states.

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1. Introduction

Miniaturization [1] and technological advancements to handle and control systems at smaller and smaller scales necessitate the deeper understanding of concepts such as quantum coherence, entanglement and correlations [2–13]. Two inequivalent resource theories of coherence have been proposed [14–16] realizing the importance of the coherence as a resource in various physical situations. Recently, it has been proved that the coherence of a random pure state sampled from the uniform Haar measure is generic for higher dimensional systems, i.e., most of the random pure states have almost the same amount of coherence [17]. The importance of this result and the similar results for entanglement of random bipartite pure states cannot be overemphasized. The average entanglement of random bipartite pure states, which is facilitated by the calculation of average entropy of the marginals of the random bipartite pure states [18–21], is proved typical [22]. This has resulted in various interesting consequences in quantum information theory [22–26], in the context of black holes [27] and in particular, in explaining the *equal a priori probability* postulate of statistical physics [28,29]. But as we approach towards the realistic implementations of quantum technology, mixed states are encountered naturally due to the interaction between the system of interest and the external world. Therefore, consideration of average entanglement and coherence content of random mixed states is of great importance in realistic scenarios. However, to the best of our knowledge, there is no known result on the average coherence of random mixed states.

Here, we aim at finding the average relative entropy of coherence of random mixed states sampled from various induced measures including the one obtained via the partial tracing of the Haar distributed random bipartite pure states. We first find the exact expression for the average subentropy of random mixed states sampled from induced probability measures and use it to find the average relative entropy of coherence of random mixed states. We note that the subentropy is a nonlinear function of state and therefore, it is expected that the average subentropy of a random mixed state should not be equal to the subentropy of the average state (the maximally mixed state). Surprisingly, we find that the average subentropy of a random mixed state approaches exponentially fast towards the maximum value of the subentropy, which is achieved for the maximally mixed state [30]. As one of the applications of our results, we note that the average subentropy may also serve as the state independent quality factor for ensembles of states to be used for estimating accessible information. Interestingly, we find that the average coherence of random mixed states, just like the average coherence of random pure states, shows the concentration phenomenon. This means that the relative entropies of coherence of most of the random mixed states are equal to some fixed number (within an arbitrarily small error) for larger Hilbert space dimensions. It is well known that the exact computation of the most of the entanglement measures for bipartite mixed states in higher dimensions is almost impossible [31]. However, using our results, we compute the average relative entropy of entanglement and distillable entanglement for a specific class of random bipartite mixed states and show their typicality for larger Hilbert space dimensions. It means that for almost all random states of this specific class, both the measures of entanglement are equal to a fixed number (that we calculate) within an arbitrarily small error, reducing hugely the computational complexity of both the measures for this specific class of bipartite mixed states. This is a very important practical application of the results obtained in this paper.

2. Quantum coherence and induced measures on the space of mixed states

2.1. Quantum coherence

Various coherence monotones, that serve as the faithful measures of coherence [16,32–34], are proposed based on the resource theory of coherence [16]. These monotones include the l_1 norm of coherence, relative entropy of coherence [16] and the geometric measure of coherence based on entanglement [32]. In this work, unless stated otherwise, by coherence we mean the relative entropy of coherence throughout the paper. The relative entropy of coherence of a quantum state ρ, acting on an m-dimensional Hilbert space, is defined as [16]: $\mathscr{C}_r(\rho) := S(\Pi(\rho)) - S(\rho)$, where

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