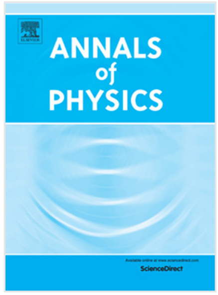

Hamiltonian models for topological phases of matter in three spatial dimensions

Dominic J. Williamson, Zhenghan Wang

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Hamiltonian models for topological phases of matter in three spatial dimensions

Dominic J. Williamson[a], Zhenghan Wang[c]

[a]Vienna Center for Quantum Technology, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria
[b]Microsoft Research Station Q, CNSI Bldg. 2237, Santa Barbara, California 93106-6105, USA
[c]Department of Mathematics, University of California, Santa Barbara, California 93106-6105, USA

Abstract

We present commuting projector Hamiltonian realizations of a large class of (3+1)D topological models based on mathematical objects called unitary G-crossed braided fusion categories. This construction comes with a wealth of examples from the literature of symmetry-enriched topological phases. The spacetime counterparts to our Hamiltonians are unitary state sum topological quantum fields theories (TQFTs) that appear to capture all known constructions in the literature, including the Crane-Yetter-Walker-Wang and 2-Group gauge theory models. We also present Hamiltonian realizations of a state sum TQFT recently constructed by Kashaev whose relation to existing models was previously unknown. We argue that this TQFT is captured as a special case of the Crane-Yetter-Walker-Wang model, with a premodular input category in some instances.

Keywords: topological order, topological phases of matter, topological quantum field theory

1. Introduction

Theoretically, a topological phase of matter (TPM)[1, 2] without any symmetry protection is an equivalence class of local Hamiltonians [3, 4, 5, 6] whose low energy physics is modeled by a stable[1] unitary topological quantum field theory (TQFT) [7, 8, 9, 10, 11, 12, 13]. Given a realistic Hamiltonian it is generally difficult to determine which TPM it is in. A fruitful approach is to reverse engineer Hamiltonians from known TQFTs. Famous examples include Kitaev's toric code [6, 14] and Levin-Wen models [15, 16].

Physical TQFTs are local and this is usually formulated by a set of axioms known as the gluing formulas [17, 18, 19]. A more explicit form of locality is a state sum construction [16]. It is generally believed that state sum TQFTs are in 1-1 correspondence with fully extended TQFTs and both admit local commuting projector Hamiltonian realizations. However this conjecture has not been rigorously proven in full generality largely due to an inability to drop restrictive symmetry assumptions on the input data and higher "j-symbols". While it is difficult to algebraically formalize the fully extended TQFT framework without these assumptions some progress has been made for the state sum case in Ref.[20]. An interesting example that clearly violates the symmetry assumptions is Kashaev's state sum $(3 + 1)$-TQFT [21, 22], whose j-symbols strongly depend on the linear ordering of the vertices of a 4-simplex.

A basic principle in the study of state sum TQFTs is that the behavior of a local $(n+1)$-TQFT restricted to a disk is encoded by some higher n-category $\mathcal{C}$ [17, 18, 19]. Furthermore the partition functions and a local commuting projector Hamiltonian can be constructed from $\mathcal{C}$ as illustrated by the Turaev-Viro and Levin-Wen models [15, 16] (generalized Kitaev models [6, 14]) in two spatial dimensions. The physical excitations in this general picture should be described by a special $(n + 1)$-category that is constructed by taking a generalized Drinfeld double of the n-category $\mathcal{C}$ [23, 24]. The major deficiencies of this general

[1]Stable can be understood as no spontaneous symmetry breaking. The technical definition is $Z[S^3 \times S^1] = 1$ which implies local operators act trivially within the ground space.