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Berry Fermi liquid theory

Jing-Yuan Chen [a,b,*], Dam Thanh Son [a]

[a] *Kadanoff Center for Theoretical Physics, University of Chicago, Chicago, IL 60637, USA*

[b] *Stanford Institute for Theoretical Physics, Stanford University, CA 94305, USA*

HIGHLIGHTS

- We extend Landau's kinetic theory of Fermi liquid to incorporate Berry phase.
- Berry phase effects in Fermi liquid take exactly the same form as in Fermi gas.
- There is a new "emergent electric dipole" contribution to the anomalous Hall effect.
- Our kinetic theory is matched to field theory to all orders in Feynman diagrams.

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ABSTRACT

We develop an extension of the Landau Fermi liquid theory to systems of interacting fermions with non-trivial Berry curvature. We propose a kinetic equation and a constitutive relation for the electromagnetic current that together encode the linear response of such systems to external electromagnetic perturbations, to leading and next-to-leading orders in the expansion over the frequency and wave number of the perturbations. We analyze the Feynman diagrams in a large class of interacting quantum field theories and show that, after summing up all orders in perturbation theory, the current–current correlator exactly matches with the result obtained from the kinetic theory.

1. Introduction

One of the main cornerstones of condensed matter physics is Landau's Fermi liquid theory. This theory, proposed by Landau in late 1950s [1,2], describes low-energy dynamics of normal Fermi

* Corresponding author at: Stanford Institute for Theoretical Physics, Stanford University, CA 94305, USA.
E-mail addresses: chjy@uchicago.edu (J.-Y. Chen), dtson@uchicago.edu (D.T. Son).

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liquids in terms of quasiparticles, whose interactions are specified by a set of Landau's parameters. The theory was constructed by Landau phenomenologically at first, and subsequently found its theoretical justification from the diagrammatic approach, where an analysis of the infrared singularities of Feynman diagrams reveals, in particular, the precise connection between the Landau parameters and the four-point vertex, evaluated in a particular kinematic regime [3–6]. More recently, Landau's Fermi liquid theory has been reinterpreted in the language of the renormalization group [7,8]. Except for a possible instability in the Bardeen–Cooper–Schrieffer (BCS) channel, Landau's Fermi liquid theory provides a truly universal low-energy effective description of Fermi system with short-ranged interactions.

It has been known for some time, however, that effects related to the Berry curvature of the fermion in momentum space are beyond the purview of Landau's theory. For reviews of these effects see Refs. [9,10]. It was shown in Ref. [11] that the semiclassical equation of motion of a wave packet should involve an anomalous velocity. One of the consequences of this modification is the anomalous Hall effect [12]. For non-interacting fermions in $(2+1)$ dimensions, the fractional part of the anomalous Hall coefficient can be related to the Berry phase that the fermion obtains when it moves around the Fermi disk [13]. Other examples include the chiral anomaly in $(3+1)$ dimensions and effects associated with it, in particular the chiral magnetic effect or the anomalous contribution to magnetoresistance [14–17].

Most treatments of fermionic systems with Berry curvature so far neglected the interactions between the fermions. This leaves one with the question: which ones of the results, obtained for non-interacting fermions, survive when interactions are taken into account? For example, does the anomalous Hall coefficient continue to be equal to the Berry phase of the quasiparticle around the Fermi disk, or there are corrections due to the interactions?

In this paper, we address the question of how the Berry phase of the fermionic quasiparticle makes appearance in the Landau's Fermi liquid theory. We derive, by resumming all Feynman diagrams, a linearized kinetic equation, capable of giving the linear electromagnetic response in a Fermi liquid with Berry curvature, up to the next-to-leading order in the expansion over momentum. This linear theory is sufficient for the anomalous Hall effect and the chiral magnetic effect. From the kinetic equation, we found that the anomalous Hall coefficient does not coincide with the Berry phase, but contains in addition to the Berry phase a contribution coming from the electric dipole moment of the quasiparticles, which we show to be in general non-zero. In this work, we limit ourselves to the linearized version of the theory. We hope to extend the theory beyond the linear order in future work.

In previous literature, the work that has most overlap with ours is Ref. [18] where the interplay between Berry curvature and interaction has been studied in a very general context. The authors of Ref. [18] showed, via the Keldysh formalism, that the quasiparticles' motion has an anomalous velocity due to the Berry curvature, as in the non-interacting case, but the content of the Berry curvature is modified by interactions. There are four main differences between Ref. [18] and this work. First, in contrast to Ref. [18], we only study linear response, which does not see the effect of the anomalous velocity in the kinetic equation. Second, in the present paper the Berry curvature effects show up in (the non-quasiparticle contribution to) the current, an effect not computed in Ref. [18]. Third, we are able to take into account the effect of the quasiparticle collisions and the finite quasiparticle lifetime. Last, we are able to answer the question whether interesting transport phenomena such as the anomalous Hall effect involve Fermi surface contribution only, or involve Fermi sea contribution as well.

The structure of the paper is as follows. In Section 2 we first review the Landau Fermi liquid theory, more precisely, the linearized version of the theory. We then propose a kinetic theory that would capture the full linear electromagnetic response of a Fermi liquid with Berry curvature. In Section 3 we show, by a careful analysis of Feynman diagrams, that the kinetic theory reproduces correctly the linear response in the field theory to all orders in perturbation theory. The analysis identifies the parameters of the kinetic theory with objects in the field theory. Section 4 contains final discussions. Appendices A and B are devoted to details about quasiparticle collisions. Appendices C and D contain certain technical details of our diagrammatic analysis.

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